ALGEBRA (LINEAR)

THIS IS PROBABLY THE MOST IMPORTANT THING TO BE ABLE TO DO IN ALGEBRA !

• SOLVING LINEAR EQUATIONS

① TIDY UP
- REMOVE ANY BRACKETS
- GET RID OF FRACTIONS
- ADD LIKE TERMS

② GET x's ON ONE SIDE [USUALLY LEFT]

③ GET NUMBERS ONTO OTHER SIDE.

④ DIVIDE BY NUMBER IN FRONT OF x TO SOLVE.

$$2(2x - 4) = 12 - 3(2x - 1)$$

① $4x - 8 = 12 - 6x + 3$

$4x - 8 = 15 - 6x$

$+6x \qquad +6x$

I DON'T WANT x's OVER HERE, SO ADD $6x$ TO BOTH SIDES

$10x - 8 = 15$

$+8 \qquad +8$

NEED TO GET RID OF THIS -8, SO ADD 8 TO BOTH SIDES

$10x = 23$

$\div 10 \qquad \div 10$

THIS MEANS "x" MULTIPLIED BY 10 SO TO UNDO THIS I DIVIDE (BOTH SIDES) BY 10

$x = 2.3$

METHOD

① TIDY UP.
- GET RID OF BRACKETS
- ADD LIKE TERMS.

② GET x's ON ONE SIDE

③ GET NUMBERS ONTO OTHER SIDE.

④ DIVIDE BY NUMBER IN FRONT OF x

WITH PRACTICE YOU WILL GET MUCH QUICKER AND WILL BE ABLE TO TAKE SHORT CUTS, BUT ONLY IF YOU UNDERSTAND WHY

QUICKER / EASIER EXAMPLES

TYPE ①

$3x = 9$

$\div 3$ $\quad$ $\div 3$

$\boxed{x = 3}$

TYPE ②

$x + 4 = 7$

$-4 \quad -4$

$\boxed{x = 3}$

TYPE ③

$2x + 7 = 17$

$-7 \quad -7$

$2x = 10$

$\div 2$ $\quad$ $\div 2$

$\boxed{x = 5}$

TYPE ④

$7x + 2 = 4x + 11$

$-4x \quad -2 \quad -4x \quad -2$

$3x = 9$

$\div 3$ $\quad$ $\div 3$

$\boxed{x = 3}$

TYPE ⑤

$3(2x + 1) = -9$

$6x + 3 = -9$

$-3 \quad -3$

$6x = -12$

$\div 6$ $\quad$ $\div 6$

$\boxed{x = -2}$

GOLDEN RULES:

- I CAN ONLY WORK ON ONE SIDE OF THE EQUATION AT ANY ONE TIME.
- WHATEVER I DO TO ONE SIDE, I HAVE TO DO THE EXACT SAME THING TO THE OTHER SIDE...

EQUATIONS WITH FRACTIONS → UGH. GET RID...

TO GET RID OF FRACTIONS FROM OUR EQUATION WE MULTIPLY EVERYTHING BY THE LOWEST COMMON DENOMINATOR ← SNAP!!! WITH NUMBERS ON BOTTOM

eg $\frac{x}{6} - \frac{x}{2} = 5$

① L.C.D. = 6 [MULTIPLY EACH TERM BY 6]

$6\left(\frac{x}{6}\right) - 6\left(\frac{x}{2}\right) = 6(5)$

② NOW DIVIDE EACH NUMBER ON THE BOTTOM INTO THE NUMBER OUTSIDE THE BRACKET.

$\cancel{6}^{1}\left(\frac{x}{\cancel{6}}\right) - \cancel{2}^{3}\left(\frac{x}{\cancel{2}}\right) = 6(5)$

③ TIDY UP

$x - 3x = 30$

$-2x = 30$ ← CHANGE SIGNS OF BOTH SIDES.

$2x = -30$

$\boxed{x = -15}$ ← DIVIDE BY 2

FACTORISING

THIS IS LIKE THE OPPOSITE OF MULTIPLYING OUT BRACKETS.

eg $(x+1)(x+3)$

$= x^2 + 4x + 3$ → $x^2 + 4x + 3$ → $(x+1)(x+3)$

2 TYPES WHICH EVERYONE FORGETS ABOUT

1) H.C.F.

- IF POSSIBLE, ALWAYS "TAKE OUT" A COMMON FACTOR. LOOK ACROSS THE EXPRESSION FOR A NUMBER/LETTER WHICH GOES INTO BOTH/ALL THE TERMS.

LOOK FOR THESE FIRST

eg $3a^2 + 9a$

$3a(a + 3)$

↑ H.C.F. (3a goes into $3a^2$ and 9a)

THIS IS WHAT YOU WOULD MULTIPLY BY THE HCF TO GET THE ORIGINAL EXPRESSION

2) DOTS

$a^2 - b^2 = (a+b)(a-b)$

DIFFERENCE OF TWO SQUARES

eg $x^2 - 81$

CAN BE WRITTEN AS $x^2 - 9^2$

$(x+9)(x-9)$

WRITE IT AS:

$(\text{SOMETHING})^2 - (\text{SOMETHING ELSE})^2$

- YOU MUST LOOK OUT FOR THESE 2 TYPES, OTHERWISE HOW WILL YOU SPOT THEM?!

INEQUALITIES

TREAT JUST LIKE EQUATIONS

BUT

NEVER CHANGE SIGNS OR MULTIPLY / DIVIDE BY A MINUS NUMBER.

$< \quad > \quad \geqslant \quad \leqslant$

[DON'T WORRY ABOUT THE SIGNS...]

eg

$$3x - 2 \leqslant 4$$

$$+2 \qquad\qquad +2$$

$$3x \leqslant 6$$

$$x \leqslant 2$$

GET x's ON ONE SIDE

GET NUMBERS ON OTHER SIDE

$\div$ BY 3

$x \leqslant 2$ LESS THAN OR EQUAL TO 2

GRAPHING INEQUALITIES

$x \in \mathbb{N}$

$\mathbb{N}$ = NATURAL NUMBERS

$x \in \mathbb{Z}$

$\mathbb{Z}$ = INTEGERS

WHOLE NUMBERS - USE DOTS

$x \in \mathbb{R}$

$\mathbb{R}$ = REAL

(USE HEAVY BLACK LINE)

SIMULTANEOUS EQUATIONS

eg
$$2x + 5y = -15$$
$$4x + 3y = -9$$

PROBLEM : • WE HAVE 2 LETTERS : x's AND y's.

• WE CAN ONLY SOLVE EQUATIONS WITH ONE LETTER.

SOLUTION : WE NEED TO GET RID OF ONE OF THE LETTERS.

GREAT... HOW? → WE NEED SAME NUMBER IN FRONT OF y's WITH 1 ⊕ AND 1 ⊖ SIGN

GREAT... HOW? → MULTIPLY EACH LINE/EQUATION BY THE NUMBER IN FRONT OF y IN THE OTHER LINE

(SOUNDS HARD – IT'S NOT)

YOU MIGHT NEED TO CHANGE THE SIGNS OF ONE OF THE EQUATIONS

THEN ...

ADD THE TWO EQUATIONS TOGETHER, AND THE y's SHOULD "DISAPPEAR"!

NEARLY THERE ... SOLVE FOR x, THEN SUBSTITUTE THIS ANSWER BACK INTO ONE OF THE ORIGINAL EQUATIONS, AND SOLVE TO FIND y!

SEE NEXT PAGE FOR EXAMPLE :

STEP ① : CALL EQUATIONS (A) + (B)

(A) $3x + 2y = 5$

(B) $4x + 5y = 2$

STEP ② : MULTIPLY EACH LINE BY NUMBER IN FRONT OF y IN OTHER LINE.

(2b) : CHANGE SIGNS OF 2ND EQUATION

$5 \times$ (A) $\quad 15x + 10y = 25$

$2 \times$ (B) $\quad -8x - 10y = -4$

STEP ③ BRILLIANT: WE HAVE SAME NUMBER OF y's AND 1 +, 1 −

SO ADD 2 EQUATIONS TOGETHER.

$7x = 21$

$\div 7$

$x = 3$

STEP ④ SOLVE FOR x

STEP ⑤ WRITE OUT ONE OF THE ORIGINAL EQUATIONS, SUBSTITUTE NEW VALUE OF x, AND SOLVE

$3x + 2y = 5$

$x = 3 \rightarrow 3(3) + 2y = 5$

$9 + 2y = 5$

$-9 \quad\quad -9$

$2y = -4$

$y = -2$

SO $x = 3$ $\quad y = -2$

WORD PROBLEMS (HARD)

- THEY GIVE YOU A "PROBLEM" IN WORDS. WE NEED TO SOMEHOW MAKE THIS INTO AN EQUATION AND SOLVE IT.

- READ QUESTION CAREFULLY

- HIGHLIGHT IMPORTANT WORDS.

- LET UNKNOWN NUMBER = x

- IF THERE ARE 2 UNKNOWNS, LET THE OTHER LETTER = y, THEN YOU WILL GET SIMULTANEOUS EQUATIONS...

- IF YOU'RE NOT SURE HOW TO FORM THE EQUATION, MAKE UP A NUMBER FOR x, AND WRITE DOWN HOW YOU WOULD WRITE DOWN THE EQUATION IF THIS WAS CORRECT.

eg Q. WHEN I MULTIPLY A NUMBER BY 12 AND ADD 37, THE RESULT IS 325. FIND THE NUMBER.

A.

NUMBER = x

PRETEND $x = 5$

MULTIPLY 5 BY 12 AND ADD 37

$5 \times 12 + 37$

BUT x IS NOT 5, SO WRITE THIS AS

$x \times 12 + 37 = 325$

SOLVE THIS → $12x + 37 = 325$