ALGEBRA (LINEAR)

• SOLVING LINEAR EQUATIONS

① TIDY UP
- REMOVE ANY BRACKETS
- GET RID OF FRACTIONS
- ADD LIKE TERMS

② GET x's ON ONE SIDE [USUALLY LEFT]

③ GET NUMBERS ONTO OTHER SIDE.

④ DIVIDE BY NUMBER IN FRONT OF x TO SOLVE.

$$2(2x - 4) = 12 - 3(2x - 1)$$

①
$$4x - 8 = 12 - 6x + 3$$
$$4x - 8 = 15 - 6x$$

$+6x$ $\qquad$ $+6x$

I DON'T WANT x's OVER HERE, SO ADD $6x$ TO BOTH SIDES

$$10x - 8 = 15$$

$+8$ $\qquad$ $+8$

NEED TO GET RID OF THIS -8, SO ADD 8 TO BOTH SIDES

$$10x = 23$$

$\div 10$ $\qquad$ $\div 10$

THIS MEANS "x" MULTIPLIED BY 10 SO TO UNDO THIS I DIVIDE (BOTH SIDES) BY 10

$$x = 2.3$$

METHOD

① TIDY UP.
- GET RID OF BRACKETS
- ADD LIKE TERMS.

② GET x's ON ONE SIDE

③ GET NUMBERS ONTO OTHER SIDE.

④ DIVIDE BY NUMBER IN FRONT OF x

WITH PRACTICE YOU WILL GET MUCH QUICKER AND WILL BE ABLE TO TAKE SHORT CUTS, BUT ONLY IF YOU UNDERSTAND WHY

GOLDEN RULES:

- I CAN ONLY WORK ON ONE SIDE OF THE EQUATION AT ANY ONE TIME.
- WHATEVER I DO TO ONE SIDE, I HAVE TO DO THE EXACT SAME THING TO THE OTHER SIDE...

EQUATIONS WITH FRACTIONS → UGH. GET RID...

TO GET RID OF FRACTIONS FROM OUR EQUATION WE MULTIPLY EVERYTHING BY THE LOWEST COMMON DENOMINATOR ← SNAP!!! WITH NUMBERS ON BOTTOM

eg $\frac{x}{6} - \frac{x}{2} = 5$

① L.C.D. = 6 ← [MULTIPLY EACH TERM BY 6]

$6\left(\frac{x}{6}\right) - 6\left(\frac{x}{2}\right) = 6(5)$

② NOW DIVIDE EACH NUMBER ON THE BOTTOM INTO THE NUMBER OUTSIDE THE BRACKET.

$\cancel{6}^{1}\left(\frac{x}{\cancel{6}}\right) - \cancel{6}^{3}\left(\frac{x}{\cancel{2}}\right) = 6(5)$

③ TIDY UP

$x - 3x = 30$

$-2x = 30$ ← CHANGE SIGNS OF BOTH SIDES.

$2x = -30$

$\boxed{x = -15}$ ← DIVIDE BY 2

SIMULTANEOUS EQUATIONS

eg
$$2x + 5y = -15$$
$$4x + 3y = -9$$

PROBLEM: • WE HAVE 2 LETTERS : x's AND y's.

• WE CAN ONLY SOLVE EQUATIONS WITH ONE LETTER.

SOLUTION: WE NEED TO GET RID OF ONE OF THE LETTERS.

GREAT.... **HOW?**

— IF WE HAVE THE SAME NUMBER OF y's IN BOTH EQUATIONS, WITH 1 (+) AND 1 (−), THEN ADDING THE EQUATIONS TOGETHER WILL LEAVE ME WITH NO y's.... GREAT!!

— **HOW?** • MULTIPLY TOP LINE BY NUMBER IN FRONT OF y ON THE BOTTOM AND VICE VERSA.

• YOU MIGHT NEED TO CHANGE THE SIGN OF ONE ENTIRE EQUATION.

So, $\times 3 \rightarrow$ ① $2x + 5y = -15$

$\times 5 \rightarrow$ ② $4x + 3y = -9$

$\times 3$ ① $6x + 15y = -45$

$\times 5$ ② $20x + 15y = -45$ ← CHANGE SIGNS OF TOP LINE

$-6x - 15y = 45$

$20x + 15y = -45$

ADD ② EQUATIONS TOGETHER... y's DISAPPEAR...

$14x = 0$

$x = 0$

CONTINUED ON NEXT PAGE...

$x = 0$ BUT WE ALSO NEED TO FIND y.

① $2x + 5y = -15$ ← CHOOSE EITHER OF YOUR FIRST TWO EQUATIONS

$2(0) + 5y = -15$ ← SUBSTITUTE IN YOUR ANSWER FOR x ($x = 0$)

$5y = -15$

$\boxed{y = -3}$

ANS: $\boxed{x = 0, \; y = -3}$

YOU SOMETIMES NEED TO RE-ARRANGE THE INITIAL EQUATIONS TO LOOK LIKE THIS...

FOR EXAMPLE,
- GET RID OF BRACKETS / FRACTIONS
- GET x's AND y's ONTO SAME SIDE
- GET NUMBERS ONTO OTHER SIDE.

IN THEORY, WHEN YOU GET AN ANSWER FOR x AND y, THIS IS A "POINT" ON THE X AND Y AXES. THIS IS THE PLACE WHERE THE 2 LINES CROSS

eg $2x + 5y = -15$ IS A LINE...

INEQUALITIES

TREAT JUST LIKE EQUATIONS BUT NEVER CHANGE SIGNS OR MULTIPLY / DIVIDE BY A MINUS NUMBER.

$< \quad > \quad \geqslant \quad \leqslant$

[DON'T WORRY ABOUT THE SIGNS...]

eg

$$3x - 2 \leqslant 4$$

$$+2 \qquad +2$$

$$3x \leqslant 6$$

$$x \leqslant 2$$

GET x's ON ONE SIDE
GET NUMBERS ON OTHER SIDE

÷ BY 3

$x \leqslant 2$ LESS THAN OR EQUAL TO 2

GRAPHING INEQUALITIES

$x \in \mathbb{N}$

$\mathbb{N}$ = NATURAL NUMBERS

$x \in \mathbb{Z}$

$\mathbb{Z}$ = INTEGERS (WHOLE NUMBERS - USE DOTS)

$x \in \mathbb{R}$

$\mathbb{R}$ = REAL (USE HEAVY BLACK LINE)

MANIPULATING FORMULAS / REARRANGING EQUATIONS WITH LOTS OF LETTERS

- THESE LOOK HARD BUT ARE ACTUALLY EASIER THAN EQUATIONS BECAUSE THERE IS NO CALCULATION INVOLVED...

eg Express t in terms of u, v, and a.

$$v = u + at$$

← SWITCHED SIDES TO GET t ON LEFT.

$$u + at = v$$

$$-u \qquad -u$$

← NEED TO GET t ON ITS OWN

$$at = v - u$$

THIS IS NEARLY IT.

$$t = \frac{v - u}{a}$$

← DIVIDE BY LETTER IN FRONT OF t

at means $t \times a$

↑ UNDO $\times a$ SO $\div a$

METHOD

WE WANT TO GET $t = ?$

1. GET ANY THING WITH t ON ITS OWN ON THE LEFT.
2. IN REALLY HARD QUESTIONS YOU WILL NEED TO TAKE OUT A FACTOR OF t.

WORD PROBLEMS (HARD)

- THEY GIVE YOU A "PROBLEM" IN WORDS. WE NEED TO SOMEHOW MAKE THIS INTO AN EQUATION AND SOLVE IT.
- READ QUESTION CAREFULLY
- HIGHLIGHT IMPORTANT WORDS.
- LET UNKNOWN NUMBER = x
- IF THERE ARE 2 UNKNOWNS, LET THE OTHER LETTER = y, THEN YOU WILL GET SIMULTANEOUS EQUATIONS...
- IF YOU'RE NOT SURE HOW TO FORM THE EQUATION, MAKE UP A NUMBER FOR x, AND WRITE DOWN HOW YOU WOULD WRITE DOWN THE EQUATION IF THIS WAS CORRECT.

eg Q. WHEN I MULTIPLY A NUMBER BY 12 AND ADD 37, THE RESULT IS 325. FIND THE NUMBER.

A.

NUMBER = x

PRETEND $x = 5$

MULTIPLY 5 BY 12 AND ADD 37

$5 \times 12 + 37$

BUT x IS NOT 5, SO WRITE THIS AS

$x \times 12 + 37 = 325$

SOLVE THIS → $12x + 37 = 325$