

ALGEBRA

INTRODUCTION

- ALGEBRA IS BRILLIANT ! (+ VERY USEFUL)
- IT IS WHERE WE USE LETTERS TO REPRESENT UNKNOWN NUMBERS
- DON'T SWITCH OFF JUST BECAUSE IT LOOKS WEIRD
- IT OBEYS ALL THE RULES OF NORMAL NUMBERS

LET'S JUST OUTLINE SOME RULES ABOUT x (OR ACTUALLY, ANY LETTER)

(1) x IS A NUMBER

(2) x COULD BE ANY NUMBER

(3) x IS THE SAME THING AS $|x$

(4) x OBEYS ALL THE USUAL RULES OF NUMBERS

(5) A NUMBER OUTSIDE A BRACKET MEANS :

MULTIPLY EVERYTHING INSIDE THE BRACKET BY THE NUMBER OUTSIDE

ALGEBRA IS USED IN ALMOST EVERY AREA OF MATHS, SO MAKE SURE YOU LEARN AND PRACTISE THIS TOPIC VERY WELL.

ALGEBRA

SUBSTITUTION

- REPLACE A LETTER WITH A NUMBER

eg if $x=3$ and $y=-2$, calculate the value of $2x+3y$

ANS

$$2x + 3y$$
$$= 2(3) + 3(-2)$$
$$= 6 - 6$$
$$= 0$$

WE REPLACE EVERY x WITH A 3
AND EVERY y WITH A -2

PUT THEM IN BRACKETS!

- FOLLOW

BIMDAS

- ① BRACKETS FIRST
THEN
- ② INDICES / POWERS
- ③ MULTIPLICATION / DIVISION
- ④ ADDITION / SUBTRACTION

SOME CONFUSION :

DON'T FORGET : IF YOU PUT IT INTO YOUR CALCULATOR CORRECTLY YOU WILL GET THE ANSWER!

- $(2)(-2)$ MEANS 2 MULTIPLIED BY -2
- $3(-1)$ MEANS 3 MULTIPLIED BY -1
- $3(2+5)$ MEANS 3 MULTIPLIED BY (2+5)
SO
3 MULTIPLIED BY 7

THIS NUMBER OF TIMES

MEANS $2 \times 2 \times 2 = 8$

- $(3) + (-2)$ MEANS ADD -2 TO 3 = 1

ADDING / SUBTRACTING ALGEBRA

- WE CAN ONLY ADD "LIKE" TERMS.

eg $3x + 2x = 5x$

KNOW WHEN TO STOP.
REALISE THAT YOU CAN'T
DO ANYTHING TO SOMETHING
LIKE $3a + 7b$

MAKE SURE
YOU FULLY "GET"
THIS PAGE.
ASK FOR HELP
BEFORE PROCEEDING
OTHERWISE!

- "LIKE" TERMS MEANS THE SAME LETTER WITH THE SAME "POWER"
- eg $4x^2 + 3x$
- ↑ NOT LIKE TERMS

- THE LETTER / POWER DOES NOT CHANGE

eg $3x + 4x$ IS
NOT $7x^2$

MULTIPLYING ALGEBRA

3 STEPS.

SIGNS ①
NUMBERS ②
LETTERS ③

IN THAT ORDER!

eg $(-2x)(3y) = -6xy$

① ② ③

JUST BECAUSE
YOU SEE A
MINUS SIGN
DOES NOT
MEAN "TAKE-
AWAY"

NOTE

$a \times a = a^2$

$a^2 \times a = a^3$

$a^5 \times a^2 = [a \times a \times a \times a \times a] \times [a \times a]$

THIS BIT IS a^5 a^2

$= a \times a \times a \times a \times a \times a \times a = a^7$

BEWARE OF THE SIGNS

SAME SIGNS = +
DIFFERENT SIGNS = -

THIS IS ONLY WHEN MULTIPLYING...

eg $[-3 - 4 = -7]$
NOT MULTIPLYING

BRACKETS

SINGLE BRACKETS



MULTIPLY EVERYTHING INSIDE THE BRACKET BY THE NUMBER/LETTER OUTSIDE

DOUBLE BRACKETS



MULTIPLY EVERYTHING IN THE FIRST BRACKET BY EVERYTHING IN THE SECOND BRACKET

- IT MIGHT HELP TO DRAW ARROWS / LINES LINKING WHAT NEEDS TO BE MULTIPLIED.

eg

$$5 \quad \overbrace{(2x - 3)}$$

OR

$$\overbrace{(x + 1)}^F \quad \overbrace{(x - 2)}^O$$

L

eg F.O.I.L.

$$(x + 2)(x + 3)$$

$$F = \text{FIRST} \times \text{FIRST} = x^2$$

$$O = \text{OUTER} \times \text{OUTER} = 3x$$

$$I = \text{INNER} \times \text{INNER} = 2x$$

$$L = \text{LAST} \times \text{LAST} = 6$$

$$= x^2 + 3x + 2x + 6$$

$$x^2 + 5x + 6$$

- IF POSSIBLE, TIDY UP WHAT'S IN THE BRACKETS FIRST.

FACTORSING

THIS IS LIKE THE OPPOSITE OF
MULTIPLYING OUT BRACKETS.

eg $(x+1)(x+3) = x^2 + 4x + 3$ ~~$\rightarrow (2+1)(2+3)$~~ $x^2 + 4x + 3$

2 TYPES WHICH EVERYONE FORGETS ABOUT

1) H.C.F.

- IF POSSIBLE, ALWAYS "TAKE OUT" A COMMON FACTOR. LOOK ACROSS THE EXPRESSION FOR A NUMBER/LETTER WHICH GOES INTO BOTH/ALL THE TERMS.

eg $3a^2 + 9a$
 $3a(a+3)$

H.C.F.

(3a goes into $3a^2$ and $9a$)

THIS IS WHAT YOU WOULD
MULTIPLY BY THE
HCF TO GET THE
ORIGINAL
EXPRESSION.

$a^2 - b^2 = (a+b)(a-b)$

2) DOTS

DIFFERENCE
OF
TWO
SQUARES

eg

$x^2 - 81$

CAN BE WRITTEN AS

$x^2 - 9^2$
 $(x+9)(x-9)$

WRITE IT AS:

$(\text{SOMETHING})^2$
 $-(\text{SOMETHING ELSE})^2$

- YOU MUST LOOK OUT FOR THESE 2 TYPES, OTHERWISE HOW WILL YOU SPOT THEM?!

FACTORISING QUADRATICS (TRIAL + ERROR METHOD)

FACTORISE : PUT IT INTO BRACKETS.

STEP 1: WRITE DOWN THE QUADRATIC EXPRESSION

eg. ① $x^2 - 3x - 10$

STEP 2: DRAW THE BRACKETS AND

② $(x - 5)(x + 2)$

STEP 3: WRITE DOWN THE 2 TERMS (USUALLY JUST x AND x) THAT MULTIPLY TO MAKE THE x^2 TERM OF THE QUADRATIC.

③ IN THIS CASE, $x \times x = x^2$

④ IN THIS CASE, TRY 5 AND 2 BECAUSE $5 \times 2 = 10$.

STEP 4: THINK OF 2 NUMBERS THAT MULTIPLY TO MAKE THE LAST TERM OF THE QUADRATIC.

STEP 5: WORK OUT THE SIGNS. (FOR THE LAST TERM)

REMEMBER!

<u>SIGNS SAME</u>	+ x + OR - x -	= +
<u>SIGNS DIFFERENT</u>	+ x - OR - x +	= -

STEP 6: MULTIPLY OUT THE BRACKETS TO CHECK IF YOU HAVE THE RIGHT ANSWER. IF YOU DON'T, GO BACK TO STEP ②.

eg. ⑥ $(x - 5)(x + 2)$ FOIL.

$$x^2 + 2x - 5x - 10$$

$$= x^2 - 3x - 10$$

STEP 7: IF YOU HAVE THE RIGHT ANSWER, REMEMBER...


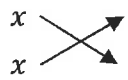

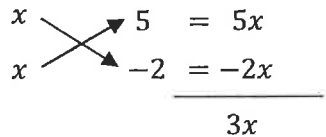
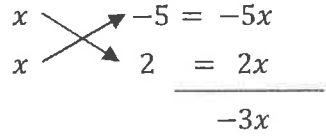
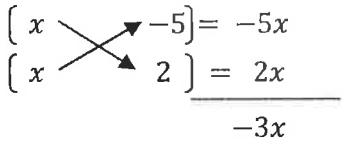
THE BRACKETS ARE THE ANSWER... NOT THE CHECKING...

↑ THIS WAS THE QUADRATIC THAT WE STARTED WITH... SO IT'S RIGHT

ANS: $(x - 5)(x + 2)$

Factorising Quadratics "Cross" Method

e.g. $x^2 - 3x - 10$

<p>Step 1: Write down the quadratic expression</p>	$x^2 - 3x - 10$
<p>Step 2: Draw in the cross underneath it, as shown</p>	
<p>Step 3: Get the FIRST term Place the x's on the left of the cross (These must multiply to make the x^2 term)</p>	$x^2 - 3x - 10$ 
<p>Step 4: Get the LAST term Place any two factors of -10 (the last term) on the right of the cross. To do this, think of any two numbers that multiply to make the last term. <i>(Remember, if the number is a minus number, the signs must be different)</i></p>	$x^2 - 3x - 10$ 
<p>Step 5 (a): Begin Trial and Error Process Multiply along the arrows. Check: do these two answers add up to the middle term? ($-3x$) If they do, move on to Step 6. If they don't, as in this example, move on to step 5 (b)</p>	$x^2 - 3x - 10$ 
<p>Step 5 (b): Try Again! Try another two factors of the last term. Multiply along the arrows. Check again: Do these two answers add up to the middle term? They do?! Great!</p>	$x^2 - 3x - 10$ 
<p>Step 6: You now have your brackets The first bracket is the top line The second bracket is the bottom line</p>	$x^2 - 3x - 10$  <p>Answer: $(x - 5)(x + 2)$</p>

DIFFICULT FACTORISING

WHEN THE x^2 TERM IS NOT $1x^2$

eg

FACTORISE $5x^2 - 16x + 3$

FOLLOW SAME STEPS AS ON THE PREVIOUS PAGE.

THIS IS WHY IT'S HARD

$5x$ x

-3 -1

TRY

$-3x$
 $-5x$

 $-8x$

X NO!
← TRY AGAIN

THE ORDER MAKES A DIFFERENCE

$5x$ x

-1 -3

$-1x$
 $-15x$

 $-16x$ ✓ YES!

NOW THE BIT WHERE EVERYONE GOES WRONG...

THE TOP LINE IS THE 1ST BRACKET $(5x - 1)$
THE BOTTOM LINE IS THE 2ND BRACKET $(x - 3)$

SO, ANSWER: $(5x - 1)(x - 3)$

ALWAYS DOUBLE-CHECK ANY FACTORISING ON A SHEET OF ROUGH WORK, BY MULTIPLYING OUT THE BRACKETS.