ALGEBRA (QUADRATICS)

- A QUADRATIC EXPRESSION HAS AN x^2 TERM (IT COULD BE A DIFFERENT LETTER)

[QUAD BIKE = 4 WHEELS
SQUARE = 4 SIDES]

eg $x^2 - 3x - 4$ IS A QUADRATIC EXPRESSION

- A QUADRATIC EQUATION HAS AN "=" SIGN.

eg $x^2 - 3x - 4 = 0$

THE "= 0" BIT IS VERY VERY IMPORTANT

IF IT'S NOT = 0 WE MIGHT NEED TO CHANGE THINGS AROUND TO GET A = 0

WE HAVE 2 WAYS TO SOLVE QUADRATIC EQUATIONS

FACTORS

BE CAREFUL OF THE SIGNS

1. MAKE SURE YOU GET AN "= 0" IN YOUR EQUATION
2. FACTORISE
3. LET EACH BRACKET = 0 AND SOLVE
4. THESE ARE YOUR 2 ANSWERS

OR

-b FORMULA

BE CAREFUL OF THE SIGNS

1. MAKE SURE YOU GET AN "= 0" IN YOUR EQUATION
2. USE THE FORMULA.
3. THESE ARE YOUR 2 ANSWERS

YOUR EQUATION SHOULD LOOK LIKE:

$ax^2 + bx + c = 0$

EITHER WAY, YOU SHOULD ALMOST ALWAYS GET 2 ANSWERS / SOLUTIONS. WE OFTEN CALL THESE THE **ROOTS** OF THE EQUATION.

FACTORISING

A EASIER ONES

1. $x^2 + 3x + 2$
2. $x^2 + 4x + 3$
3. $x^2 + 6x + 5$
4. $x^2 + 8x + 7$
5. $x^2 + 12x + 11$
6. $x^2 + 6x + 8$
7. $x^2 + 5x + 4$
8. $x^2 + 7x + 12$
9. $x^2 + 7x + 10$
10. $x^2 + 11x + 10$
11. $x^2 + 8x + 12$
12. $x^2 + 13x + 12$
13. $x^2 - 9x + 14$
14. $x^2 - 10x + 21$
15. $x^2 - 8x + 12$
16. $x^2 - 2x - 8$
17. $x^2 + 8x - 20$
18. $x^2 - 4x - 12$
19. $x^2 + 2x - 15$
20. $x^2 - x - 12$
21. $x^2 + x - 30$

BE CAREFUL

WATCH THOSE SIGNS !!

B HARDER ONES

1. $2x^2 + 5x + 3$
2. $3x^2 + 8x - 3$
3. $2x^2 - 7x + 6$
4. $3x^2 - 4x - 7$
5. $2x^2 - 9x - 5$
6. $5x^2 + 9x - 2$
7. $2x^2 + 7x - 15$
8. $3x^2 - 11x - 20$
9. $7x^2 + 5x - 2$

EXAMPLES

A 8. $x^2 + 7x + 12$

$x \times 3 = 3x$

$x \times 4 = 4x$

$3x + 4x = 7x$ ✓

$(x + 3)(x + 4)$

B 4. $3x^2 - 4x - 7$

$3x \times -7 = -7x$

$x \times 1 = 1$

$-7x + 3x = -4x$ ✓

$(3x - 7)(x + 1)$

FACTORISING TO SOLVE A QUADRATIC EQUATION

eg $2x^2 + 15x + 7 = 0$

① MAKE SURE EQUATION HAS $=0$

$2x \times 1 = 1x$

$x \times 7 = 7x$

$1x + 7x = 15x$ ✓

② FACTORISE

$(2x + 1)(x + 7) = 0$

③ LET EACH FACTOR $=0$ + SOLVE

$2x + 1 = 0$

$-1 \quad -1$

$2x = -1$

$x = \frac{-1}{2}$

$x + 7 = 0$

$x = -7$

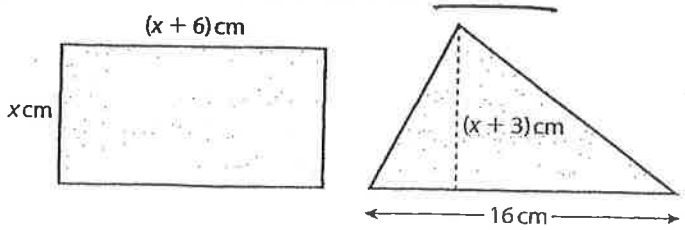
2 ROOTS

THEY SOMETIMES LIKE TO HIDE THE QUADRATIC EQUATION, FOR EXAMPLE BY GIVING YOU A RECTANGLE / TRIANGLE AND ASKING YOU TO WORK OUT THE AREA. REMEMBER, ALGEBRA FOLLOWS THE SAME RULES AS NORMAL NUMBERS!

YOU WILL NEED TO RECOGNISE QUADRATIC EQUATIONS IN THESE UNFAMILIAR SITUATIONS.

SEE THE NEXT PAGE FOR AN EXAMPLE...

EG. The rectangle and triangle below each have the same area.



(i) Write an expression in x for
(a) the area of the rectangle (b) the area of the triangle.

(ii) Form an equation and solve it to find the value of x.
Hence find the dimensions of the rectangle.
Why did you take only one value for x?

(i) (a) LENGTH × WIDTH

$x(x+6)$

$= x^2 + 6x$

(b) $\frac{1}{2}$ BASE × $\perp$ HEIGHT

$= \frac{1}{2}(16)(x+3)$

$= 8(x+3)$

$= 8x + 24$

(ii) SAME AREA. SO

$x^2 + 6x = 8x + 24$

$-8x \quad -24 \quad -8x \quad -24$

WE NEED AN $= 0$

$x^2 - 2x - 24 = 0$

$x \quad 4 \quad 4x$

$x \quad -6 \quad -6x$

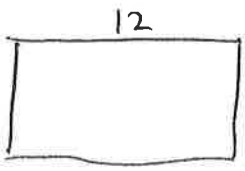
$-2x$ ✓

$(x + 4)(x - 6) = 0$

$x \neq -4 \quad x = 6$

RULE OUT $x = -4$ BECAUSE A SIDE OF A RECTANGLE MUST BE A POSITIVE NUMBER.

RECTANGLE =



THEY CAN ALSO HIDE QUADRATIC EQUATIONS IN EQUATIONS WITH FRACTIONS. FOLLOW ALL THE USUAL RULES FOR EQUATIONS WITH FRACTIONS.

YOU WILL NEED TO RECOGNISE A QUADRATIC EQUATION

eg $\dfrac{x+7}{3} + \dfrac{2}{x} = 4$

L.C.M. $= 3x$
SO MULTIPLY EACH TERM BY $3x$

$$3x\left(\frac{x+7}{3}\right) + 3x\left(\frac{2}{x}\right) = 3x\left(4\right)$$

$$x^2 + 7x + 6 = 12x$$
$$-12x \qquad -12x$$

WE NEED AN $= 0$

$$x^2 - 5x + 6 = 0$$

NOW SOLVE IN THE USUAL WAY.

SEE SECTION ON LINEAR EQUATIONS FOR OTHER EXAMPLES OF EQUATIONS WITH FRACTIONS

QUADRATIC FORMULA

- THE **OTHER METHOD** TO SOLVE QUADRATIC EQUATIONS.

- YOU NEED TO KNOW THIS, AS IT CAN SOMETIMES COME UP IN THE "COMPLEX NUMBERS" QUESTION. IT **CAN ALSO BE THE ONLY WAY** TO ANSWER CERTAIN QUADRATIC EQUATIONS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

SOME NOTES

1. THE FORMULA IS ON THE FRONT COVER OF THE TABLES BOOK!
2. BE CAREFUL WITH SIGNS.
3. YOU WILL GET 2 ANSWERS
4. $\pm$ MEANS "PLUS OR MINUS"

STEPS

① WRITE EQUATION IN CORRECT ORDER

eg $ax^2 + bx + c = 0$

↑ IMPORTANT.

② WRITE DOWN WHAT $a=$, $b=$, $c=$.
(THESE TAKE WHATEVER SIGN IS ATTACHED TO THEM IN THE EQUATION)

eg $3x^2 - 7x + 2 = 0$

$a = 3 \quad b = -7 \quad c = 2$

③ WORK OUT YOUR 2 ANSWERS. USE A CALCULATOR, BUT PLEASE... **WATCH THE SIGNS**

- IT CAN BE EASIER IF YOU DO THE $\sqrt{}$ BIT FIRST, ON SOME ROUGH WORK

EXAMPLE ON NEXT PAGE...

eg $3x^2 + 5x - 3 = 0$

$a = 3 \quad b = 5 \quad c = -3$

$\pm$ MEANS YOU WILL GET 2 ANSWERS

1 ⊕ / 1 ⊖

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-5 \pm \sqrt{(5)^2 - 4(3)(-3)}}{2(3)}$$

RW

$\sqrt{(5)^2 - 4(3)(-3)} = \sqrt{61}$

$$\frac{-5 \pm \sqrt{61}}{6}$$

$$\frac{-5 + \sqrt{61}}{6} \quad \text{OR} \quad \frac{-5 - \sqrt{61}}{6}$$

$x = 0.468$ OR $x = -2.135$

QUADRATIC EQUATIONS (WORKING BACKWARDS)

- SOMETIMES YOU WILL BE GIVEN THE ANSWERS / ROOTS AND BE ASKED TO FORM THE EQUATION.

FIRSTLY, LET'S LOOK AT WHAT THE "NORMAL" QUADRATIC EQUATION LOOKS LIKE

EQUATION → $x^2 - 6x + 5 = 0$

$x \times x$, -5×-1, $-5x - 1x = -6x$ ✓

BRACKETS → $(x - 5)(x - 1) = 0$

$x - 5 = 0$, $+5 \quad +5$, $x = 5$

$x - 1 = 0$, $+1 \quad +1$, $x = 1$

ROOTS → $x = 5$ $\quad x = 1$

NOW WE NEED TO GO THE OTHER WAY.

SO, ROOTS → BRACKETS

BRACKETS → EQUATION

EG. FORM THE QUADRATIC EQUATION WITH THE ROOTS 3 AND -1

ROOTS → $x = 3$, $x_{-3} = 3_{-3}$, $x - 3 = 0$ $\quad$ $x = -1$, $x_{+1} = -1_{+1}$, $x + 1 = 0$

THIS MEANS $x = 3$ $\quad x = -1$

← WE NEED $= 0$

THESE ARE NOW THE BRACKETS

BRACKETS → $(x - 3)(x + 1) = 0$

FOIL

$x^2 + 1x - 3x - 3 = 0$

EQUATION → $x^2 - 2x - 3 = 0$

NIGHTMARE

SIMULTANEOUS EQUATIONS WITH 1 QUADRATIC.

eg SOLVE: $x^2 + y^2 = 10$ → QUADRATIC

$x - y = 4$ → LINEAR

- THESE ARE LONG, BUT CAN COME UP IN DIFFERENT PARTS OF THE COURSE, SO WE HAVE TO BE ABLE TO DO THEM...

- WE ARE BEING ASKED TO FIND THE **POINTS OF INTERSECTION** OF THE TWO FUNCTIONS / EQUATIONS.

- YOU WILL NEED TO DO A FEW BITS OF ROUGH WORK AT TIMES.

- IN THE MIDDLE, YOU WILL GET A QUADRATIC EQUATION. YOU CAN SOLVE THIS USING EITHER FACTORISING OR THE $-b$ FORMULA.

- SEE THE EXAMPLE ON THE NEXT PAGE.

METHOD

1. WRITE EQUATIONS
2. ISOLATE ONE OF THE LETTERS FROM THE LINEAR.
3. SUBSTITUTE THIS INTO THE QUADRATIC.
4. THIS GIVES YOU A QUADRATIC EQUATION WITH 1 LETTER
5. SOLVE [2 ANSWERS]
6. DON'T FORGET TO GET THE OTHER LETTER IN EACH CASE...

① $x^2 + y^2 = 10$

② $x - y = 4$

② $x - y = 4$

$+y \quad +y$

ISOLATE ONE OF THE LETTERS

$x = 4 + y$ ③

SUB THIS INTO QUADRATIC.

① $x^2 + y^2 = 10$

$(4+y)^2 + y^2 = 10$

$16 + 8y + y^2 + y^2 = 10$

$-10 \qquad -10$

TIDY UP TO GET = 0 → $2y^2 + 8y + 6 = 0$ ← THIS IS YOUR NEW QUADRATIC EQUATION

↑ RW

$(4+y)^2$

$(4+y)(4+y)$

$4(4+y) + y(4+y)$

$16 + 4y + 4y + y^2$

$16 + 8y + y^2$

METHOD 1 FACTORS

$2y^2 + 8y + 6 = 0$

$\div 2$ $y^2 + 4y + 3 = 0$

$y \times 3$

$y \times 1$

$(y+3)(y+1) = 0$

$y = -3$ OR $y = -1$

$x = 4 + y$ ③ | $x = 4 + y$ ③

$x = 4 - 3 = 1$ | $x = 4 - 1 = 3$

$x = 1$ | $x = 3$

$(1, -3)$ | $(3, -1)$

METHOD 2 −b FORMULA

$a = 2$

$b = 8$

$c = 6$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-8 \pm \sqrt{(8)^2 - 4(2)(6)}}{2(2)}$$

$$\frac{-8 \pm \sqrt{64 - 48}}{4}$$

$$\frac{-8 + \sqrt{16}}{4} \quad \text{OR} \quad \frac{-8 - 4}{4}$$

$y = -1$ | OR $y = -3$

$x = 4 - 1$ | $x = 4 - 3$

$x = 3$ | OR $x = 1$

$(3, -1)$

IT CAN HELP TO DRAW A GRAPH

OR

YOU MIGHT BE ASKED TO USE A GRAPH TO SOLVE QUADRATIC EQUATIONS.

VERY IMPORTANT

"ROOTS" = ANSWERS = WHERE GRAPH CROSSES x-AXIS

WHERE 2 GRAPHS CROSS / TOUCH ARE CALLED "POINTS OF INTERSECTION"

METHOD

- EITHER DRAW THE GRAPH OR USE THE GRAPH GIVEN TO YOU.
- IDENTIFY THE "ROOTS" / "SOLUTIONS" / "ANSWERS" BY FINDING WHERE THE GRAPH CROSSES THE x-AXIS.
- THIS IS AN ESTIMATE - YOU MAY BE ASKED TO SUBSTITUTE YOUR ANSWER BACK IN TO THE ORIGINAL FUNCTION: REPLACE THE x IN THE FUNCTION WITH YOUR ANSWER(S) FOR x.