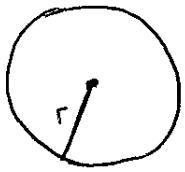
THE CIRCLE

YOU WILL NEED TO KNOW :



CENTRE

r = radius

① EQUATION OF THE CIRCLE

2 OPTIONS

EASY → CENTRE $(0,0)$

MEDIUM → OTHER CENTRE (h, k)

EQUATION :

$$x^2 + y^2 = r^2$$

$$(x - h)^2 + (y - k)^2 = r^2$$

NOTE :
- THE EQUATION OF THE CIRCLE ALWAYS HAS x's AND y's IN IT. ALWAYS
- DON'T REPLACE THE x OR y UNLESS YOU ARE CHECKING IF A POINT IS IN/ON/OUTSIDE

YOU NEED TO BE ABLE TO VERY QUICKLY / EASILY
- WRITE THE EQUATION OF A CIRCLE
- RECOGNISE THE CENTRE + RADIUS IF GIVEN THE EQUATION

eg.

1) CENTRE $(0,0)$ RADIUS $= 4$ $\Rightarrow$ EQUATION : $x^2 + y^2 = 16$

2) CENTRE $(2,-3)$ RADIUS $= 3$ $\Rightarrow$ EQUATION : $(x-2)^2 + (y+3)^2 = 9$

3) EQUATION : $x^2 + y^2 = 25$ $\Rightarrow$ CENTRE $= (0,0)$, RADIUS $= \sqrt{25} = 5$

4) EQUATION : $(x + 1)^2 + (y - 4)^2 = 49$ $\Rightarrow$ CENTRE $= (-1, 4)$, RADIUS $= 7$

HARDER EQUATION QUESTIONS :

- THEY MIGHT TRY TO "HIDE" THE IMPORTANT INFORMATION FROM YOU.
- YOU NEED TO KNOW
  - THE CENTRE
  - THE RADIUS
- IF YOU ARE STUCK... 1) DRAW THE DIAGRAM USING SQUARED PAPER
  2) USE DISTANCE FORMULA OR PYTHAGORAS OR WORK OUT MID-POINT.

REMEMBER IF THEY TELL YOU TWO POINTS ARE EITHER END OF A DIAMETER, THE CENTRE IS HALF-WAY BETWEEN THEM.

DON'T GET CONFUSED IF THEY GIVE YOU A PICTURE WITH MORE THAN ONE CIRCLE !

CHECKING IF A POINT IS IN / ON / OUTSIDE A CIRCLE :

IF A POINT IS ON THE CIRCLE, IT SATISFIES THE EQUATION i.e. IT WILL BE TRUE WHEN YOU SUBSTITUTE IN THE POINT.

eg. IS THE POINT $(3, 4)$ ON THE CIRCLE $x^2 + y^2 = 25$

METHOD

CHECK BY SUBSTITUTING INTO THE LEFT OF THE EQUATION

x y

$(3, 4)$ $\longrightarrow$ $x^2 + y^2 = 25$

$3^2 + 4^2$

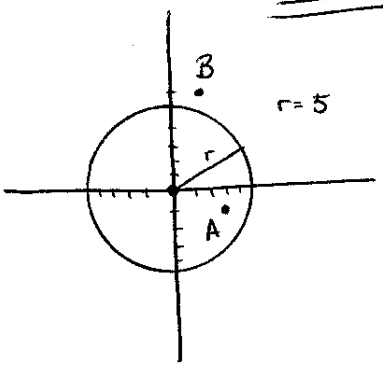
$9 + 16$

$25 = 25$ TRUE $\Rightarrow$ ON THE CIRCLE

CAUTION

- DON'T GO REPLACING x's AND y's IN THE EQUATION OF EVERY CIRCLE YOU HAVE EVER SEEN
- ONLY SUBSTITUTE IN A POINT IF YOU ARE CHECKING / VERIFYING IF THE POINT IS ON / INSIDE / OUTSIDE THE CIRCLE.

IN / OUT / ON



EQUATION: $x^2 + y^2 = 25$

$A = (3,-1) \rightarrow (3)^2 + (-1)^2 = 10 \quad 10 < 25 \Rightarrow$ INSIDE

$B = (2,6) \rightarrow (2)^2 + (6)^2 = 40 \quad 40 > 25 \Rightarrow$ OUTSIDE

- ONLY SUBSTITUTE INTO THE LEFT HAND SIDE
- COMPARE THIS TO THE RIGHT.

THIS IS THE HARD BIT : (BUT VERY IMPORTANT) KEEP REVISITING THIS PAGE OVER AND OVER

INTERSECTION OF A LINE AND CIRCLE.

eg ① $x - y = 1$

② $x^2 + y^2 = 13$

← TWO EQUATIONS

← THE PROBLEM IS, THEY BOTH HAVE x's AND y's

SO WE NEED TO USE THE SIMPLER EQUATION TO ELIMINATE A LETTER (THE y's)

① $x - y = 1$

$+y \quad +y$

③ $\boxed{x = 1 + y}$

← WAHEY !! WE NOW HAVE x ON ITS OWN.

SUBSTITUTE THIS INTO THE CIRCLE ②

② $x^2 + y^2 = 13$

$(1+y)^2 + y^2 = 13$ ← REPLACE x WITH $(1+y)$

$1 + 2y + y^2 + y^2 = 13$

NOW WE NEED TO TIDY THIS UP :

RW $\boxed{(1+y)^2}$

$= (1+y)(1+y)$

$= 1 + 1y + 1y + y^2$

$= \boxed{1 + 2y + y^2}$

$2y^2 + 2y + 1 = 13$

$-13 \quad -13$

← NEED TO GET = 0

$2y^2 + 2y - 12 = 0$

DIVIDE EVERYTHING BY THIS ÷2

$y^2 + y - 6 = 0$

← THIS IS NOW A QUADRATIC EQUATION...

$y \times 3 = 3y$

$y \times -2 = -2y$

$1y$ ✓

$(y+3)(y-2) = 0$

$\boxed{y = -3}$ | $\boxed{y = 2}$

← WE HAVE 2 ANSWERS FOR y

WE NOW NEED THE CORRESPONDING VALUES OF x

USE ③ FROM UP ABOVE

$x = 1 + y$

$x = 1 + y$ | $x = 1 + y$

$x = 1 - 3$ | $x = 1 + 2$

$\boxed{x = -2}$ | $\boxed{x = 3}$

$(-2, -3)$ | $(3, 2)$

← THESE ARE YOUR ANSWERS.

WELL DONE.

WHERE DOES THE CIRCLE INTERSECT THE X-AXIS OR THE Y-AXIS ?

2 METHODS

SOMETIMES, IF WE DRAW THE CIRCLE, WE CAN EASILY READ / SEE WHERE THE CIRCLE INTERSECTS THE X-AXIS OR Y-AXIS

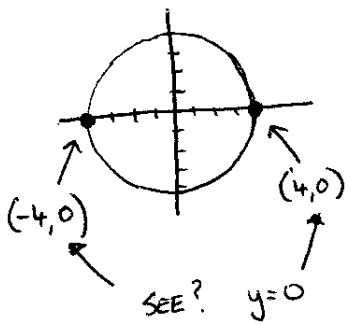
REMEMBER,
ANYWHERE ON THE X-AXIS, $y = 0$
ANYWHERE ON THE Y-AXIS, $x = 0$

← LEARN THIS

SO SUBSTITUTE INTO THE EQUATION OF THE CIRCLE, THEN SOLVE FOR THE OTHER LETTER.

eg. ① WHERE DOES $x^2 + y^2 = 16$ CROSS THE X-AXIS ?

← TRY DRAWING IT



② WHERE DOES $(x+3)^2 + (y-2)^2 = 10$ CROSS THE Y-AXIS ?

CROSS THE Y-AXIS $\Rightarrow$ $x = 0$

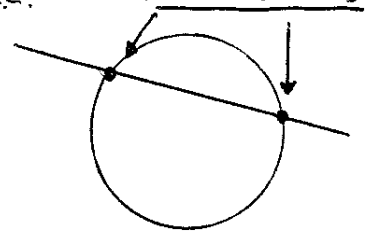
$$(0+3)^2 + (y-2)^2 = 10$$
$$9 + (y-2)^2 = 10$$
$$9 + y^2 - 4y + 4 = 10$$
$$-10 \quad\quad -10$$
$$y^2 - 4y + 13 - 10 = 0$$
$$y^2 - 4y + 3 = 0$$
$$(y-1)(y-3) = 0$$

$y = 1 \quad y = 3$

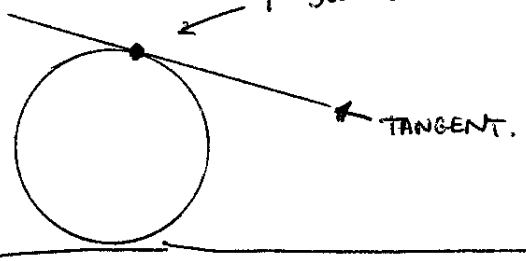
TANGENTS / POINTS OF INTERSECTION

IF YOU SOLVE ONE OF THOSE REALLY LONG QUESTIONS, WHAT YOU HAVE ARE THE POINTS OF INTERSECTION OF A LINE AND A CIRCLE

i.e. THESE 2 POINTS



IF YOU ONLY GET 1 ANSWER, WHAT YOU HAVE IS A TANGENT



- YOU MIGHT BE ASKED TO PROVE THAT A LINE IS A TANGENT.
  - → USE THE METHOD FROM THE PREVIOUS PAGE
  - → YOU WILL GET ONLY 1 ANSWER.
  - → SO, LINE TOUCHES CIRCLE AT ONLY 1 POINT
  - → SO IT'S A TANGENT!

CIRCLE TIPS

- IF IN DOUBT, DRAW IT OUT

- TO WRITE THE EQUATION OF THE CIRCLE YOU NEED 2 THINGS → RADIUS
  → CENTRE

- DON'T GET SCARED IF THEY GIVE YOU 2 OR MORE CIRCLES IN ONE PICTURE.

- THIS TOPIC IS VERY CLOSELY LINKED TO THE LINE, SO MAKE SURE YOU UNDERSTAND IT AS WELL.