COMPLEX NUMBERS

eg $\underset{\substack{\uparrow \\ \text{REAL}}}{3} + \underset{\substack{\uparrow \\ \text{IMAGINARY}}}{4i}$ OR $\underset{\substack{\uparrow \\ \text{REAL}}}{a} + \underset{\substack{\uparrow \\ \text{IM}}}{bi}$

JUST LIKE ALGEBRA - EXCEPT $\boxed{i^2 = -1}$

OR

$\boxed{i = \sqrt{-1}}$

INTRODUCTION

- WE NEED COMPLEX NUMBERS TO DEAL WITH SQUARE ROOTS OF NEGATIVE NUMBERS.
- TRY ENTERING $\sqrt{-4}$ INTO YOUR CALCULATOR -> YOU WILL GET AN ERROR.
- WE HAVE $\boxed{i = \sqrt{-1}}$. SO

$$\boxed{\sqrt{-4}} = \underset{\substack{\uparrow \\ \sqrt{4}=2}}{\sqrt{4}} \times \sqrt{-1} = \boxed{2i}$$

- IN PRACTICE, A LOT OF WHAT WE DO WITH COMPLEX NUMBERS IS JUST NORMAL ALGEBRA EXCEPT :

EVERYWHERE YOU COME ACROSS i^2

REPLACE IT WITH -1

SO $3i^2 = -3$

$-5i^2 = +5$

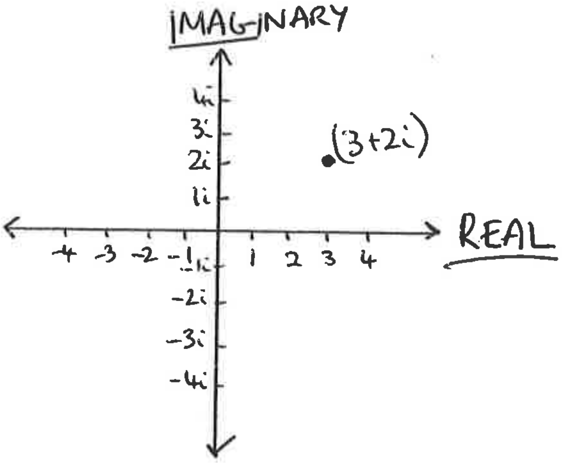
HINT :

IF YOU SEE i^2, CHANGE THE SIGN AND DROP THE i^2

ARGAND DIAGRAM

THIS IS WHERE WE PLOT COMPLEX NUMBERS AS CO-ORDINATES. (LIKE X AND Y AXES)

IT OFTEN HELPS US TO DRAW A PICTURE.



MODULUS

$|z|$

(THIS IS THE DISTANCE FROM THE "ORIGIN" TO THE "POINT" REPRESENTED ON THE ARGAND DIAGRAM).

BE VERY FAMILIAR WITH THE NOTATION / HOW THE MODULUS IS WRITTEN ...

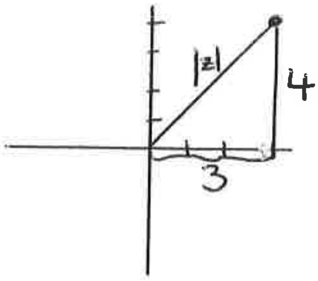
① PLOT THE POINT ON THE ARGAND DIAGRAM. (THIS CAN BE A VERY QUICK SKETCH)

② DRAW A RIGHT-ANGLED TRIANGLE + LABEL SIDE LENGTHS

③ USE PYTHAGORAS

eg $z = 3 + 4i$

CALCULATE $|z|$ →



$$|z|^2 = 3^2 + 4^2$$

$$|z|^2 = 9 + 16$$

$$|z|^2 = 25$$

$$|z| = \sqrt{25} = \boxed{5}$$

· WE OFTEN USE THE LETTERS Z OR W TO REPRESENT COMPLEX NUMBERS.

eg $Z = 3 + 2i$

WHAT IS Z^2

ANS:
$$(3+2i)^2$$
$$= (3+2i)(3+2i)$$
$$= 9 + 6i + 6i + 4i^2$$
$$= 9 + 12i - 4$$
$$= \boxed{5 + 12i}$$

CONJUGATE

$\bar{Z}$

YOU HAVE TO KNOW THE NOTATION

eg IF $Z = 3 + 5i$

$$\downarrow$$

$\bar{Z} = 3 - 5i$

CHANGE THE SIGN OF THE IMAGINARY PART ONLY

eg ① $Z = 3 + 4i$

$\bar{Z} = 3 - 4i$

② $w = 2 - 5i$

$\bar{w} = 2 + 5i$

* THIS WILL BECOME VERY USEFUL FOR DIVIDING COMPLEX NUMBERS

INVESTIGATION : WHAT HAPPENS WHEN YOU MULTIPLY BY i ?

$i = \boxed{i}$

$i^2 = i \times i = i^2 = \boxed{-1}$

$i^3 = i \times i \times i$

$= (i \times i) \times i$

$= -1 \times i = \boxed{-i}$

$i^4 = i^3 \times i$

$= (-i) \times i = -i^2 = \boxed{1}$

$i^5 = i^4 \times i$

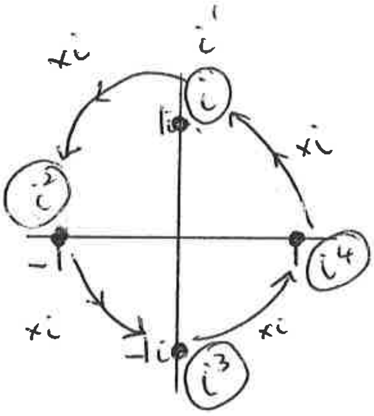
$= 1 \times i = \boxed{i}$

etc.

$$\left[\begin{array}{l} i = i \\ i^2 = -1 \\ i^3 = -i \\ i^4 = 1 \end{array}\right]$$

REPEATS EVERY FOUR

$$\left[\begin{array}{l} i^5 = i \\ i^6 = -1 \\ i^7 = -i \\ i^8 = 1 \end{array}\right]$$



SO, MULTIPLYING BY i IS A <u>ROTATION</u> BY 90°

↑

YOU SHOULD KNOW/ REMEMBER THIS ...

ADDING / SUBTRACTING COMPLEX NUMBERS

JUST LIKE IN ALGEBRA, WHEN WE CAN ONLY ADD "LIKE TERMS", WITH COMPLEX NUMBERS WE CAN ADD/SUBTRACT "REALS" AND "IMAGINARIES" SEPARATELY:

eg $3 + 4i + 2 - 3i$

REAL

$3+2$

IMAGINARY

$4i - 3i$

$= \boxed{5 + 1i}$

MULTIPLYING COMPLEX NUMBERS

BEHAVES EXACTLY LIKE ALGEBRA, EXCEPT $i^2 = -1$

EG.

① $3(4 - 5i)$

$= 12 - 15i$

② $2(3+2i) - 4(2+i)$

$= 6 + 4i - 8 - 4i$

$= \boxed{-2}$

③ $3i(1+2i)$

$= 3i + 6i^2$

REMEMBER, $\boxed{i^2 = -1}$

$= 3i - 6$

$= -6 + 3i$ ← WE WRITE THE REAL BIT FIRST

④ $(-3+2i)(4-5i)$

FOIL

$= -12 + \underbrace{15i + 8i} - 10i^2$

$= -12 + 23i + 10$

$= -2 + 23i$

DIVIDING COMPLEX NUMBERS

eg $\frac{3+2i}{4-5i}$

① EASY !

IF IT'S JUST A REAL NUMBER ON THE BOTTOM.

→ DIVIDE EACH TOP TERM BY THE NUMBER ON THE BOTTOM

eg. ① $\frac{10+15i}{5}$

$= \boxed{2+3i}$

② $\frac{23-17i}{4}$

$= \boxed{\frac{23}{4} - \frac{17}{4}i}$

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② HARD

IF THE BOTTOM HAS AN IMAGINARY PART → MAKE IT INTO THE "EASY" TYPE

eg $\frac{5+5i}{1+2i}$

CONJUGATE OF $1+2i$

$\downarrow$

$(1-2i)$

STEP 1 MULTIPLY TOP AND BOTTOM BY $(1-2i)$

$$\frac{(5+5i)(1-2i)}{(1+2i)(1-2i)}$$

STEP 2 LAY IT OUT LIKE THIS

TOPS

$(5+5i)(1-2i)$

$5(1-2i) + 5i\,(1-2i)$

$5 - 10i + 5i - 10i^2$

$5 - 5i + 10$

TOP : $\boxed{15 - 5i}$

BOTS

$(1+2i)(1-2i)$

$1(1-2i) + 2i\,(1-2i)$

$1 - 2i + 2i - 4i^2$

$1 + 4$

BOT : $\boxed{5}$

STEPS 3 + 4 : PUT $\frac{\text{TOP}}{\text{BOT}}$ AND DIVIDE

$$\frac{15-5i}{5} = \boxed{3 - i}$$

HOW ?

BY MULTIPLYING TOP AND BOTTOM BY THE CONJUGATE OF THE BOTTOM

eg

COMPLEX NO.	CONJUGATE
$4+3i$	$4-3i$
$-2-4i$	$-2+4i$
$5-7i$	$5+7i$

ie CHANGE THE SIGN OF THE IMAGINARY PART

$\downarrow$

WHY ?

BECAUSE IT MAKES THE BOTTOM A REAL NUMBER

USING COMPLEX NUMBERS FOR QUADRATIC EQUATIONS

eg $z^2 - 6z + 34 = 0$

$a = 1$

$b = -6$

$c = 34$

$$\boxed{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

THIS CAN'T BE SOLVED USING **REAL** NUMBERS ALONE!

SUB IN

$$\boxed{\frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(34)}}{2(1)}}$$

RW DO THE BIT UNDER THE SQUARE ROOT SIGN SEPARATELY.

$$\sqrt{(-6)^2 - 4(1)(34)}$$

$$= \sqrt{36 - 136}$$

$$= \sqrt{-100}$$

$$= \sqrt{100} \times \sqrt{-1}$$

$$= \boxed{10 \quad i}$$

$$\frac{6 \pm 10i}{2}$$

$\boxed{3 + 5i}$ OR $\boxed{3 - 5i}$

THIS WILL ALWAYS HAPPEN. IF THE 2 ANSWERS ARE COMPLEX NUMBERS, ONE WILL BE THE "CONJUGATE" OF THE OTHER

YOU WILL NEED TO PRACTISE LOTS OF THESE

eg ① $z^2 - 10z + 29 = 0$

② $z^2 + 2z + 10 = 0$

③ $z^2 - 12z + 37 = 0$

④ $z^2 - 2z + 17 = 0$