**Complex Numbers**

*Example:* $3 + 4i$  
*Except:* $a + bi$

**Real**  
**Imaginary**

**Just Like Algebra — Except**  
$c^2 = -1$

**OR**

$c = i\sqrt{-1}$

**Introduction**

- **We Need Complex Numbers to Deal with Square Roots of Negative Numbers.**

- **Try Entering** $\sqrt{-4}$ **Into Your Calculator** → You Will Get an Error.

- **We Have** $i = \sqrt{-1}$. **So**

$$\sqrt{-4} = \sqrt{4} \times \sqrt{-1} = 2i$$

**In Practice,** a lot of what we do with complex numbers is just normal algebra except:

**Everywhere you come across** $i^2$  
**Replace it with** $-1$

**So** $3i^2 = -3$  
$-5i^2 = +5$

**Hint:**

If you see $i^2$, change the sign and drop the $i^2$. 
ARGAND DIAGRAM

It often helps us to draw a picture.

This is where we plot complex numbers as coordinates (like x and y axes).

MODULUS (This is the distance from the "origin" to the "point" represented on the Argand diagram)

Be very familiar with the notation/how the modulus is written...

1. Plot the point on the Argand diagram. (This can be a very quick sketch)
2. Draw a right-angled triangle & label side lengths
3. Use Pythagoras

eg \[ z = 3 + 4i \]
Calculate \[ |z| \]

\[ |z|^2 = 3^2 + 4^2 \]
\[ |z|^2 = 9 + 16 \]
\[ |z|^2 = 25 \]
\[ |z| = \sqrt{25} = 5 \]
We often use the letters \( z \) or \( w \) to represent complex numbers.

**Example:**

\[ z = 3 + 2i \]

What is \( z^2 \)?

**Answer:**

\[
\begin{align*}
(3+2i)^2 &= (3+2i)(3+2i) \\
&= 9 + 6i + 6i + 4i^2 \\
&= 9 + 12i - 4 \\
&= 5 + 12i
\end{align*}
\]

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**Conjugate**

**Example:**

If \( z = 3 + 5i \)

\[ \overline{z} \]

You have to know the notation.

\[ \overline{z} = 3 - 5i \]

**Change the sign of the imaginary part only**

**Example:**

1. \( z = 3 + 4i \)
   \[ \overline{z} = 3 - 4i \]
2. \( w = 2 - 5i \)
   \[ \overline{w} = 2 + 5i \]

*This will become very useful for dividing complex numbers.*
INVESTIGATION: WHAT HAPPENS WHEN YOU MULTIPLY BY i?

\[ i = 1 \]
\[ i^2 = i \times i = i^2 = -1 \]
\[ i^3 = i \times i \times i = (i \times i) \times i = -i \times i = -i \]
\[ i^4 = i^3 \times i = (-i) \times i = -i^2 = 1 \]
\[ i^5 = i^4 \times i = 1 \times i = i \]

\[ \begin{array}{c|c|c|c|c|c}
   i^0 & i^1 & i^2 & i^3 & i^4 & i^5 \\
   \hline
   1 & i & -1 & -i & 1 & i \\
   \hline
   1 & 0 & 0 & 0 & 0 & 0
\end{array} \]

REPEATS EVERY FOUR

So, multiplying by i is a rotation by 90°.

YOU SHOULD KNOW/REMEMBER THIS...
**Adding/Subtracting Complex Numbers**

Just like in algebra, when we can only add "like terms", with complex numbers we can add/subtract "reals" and "imaginaries" separately:

\[
\begin{align*}
3 + 4i + 2 - 3i &= \boxed{5 + i} \\
\end{align*}
\]

**Multiplying Complex Numbers**

Behaves exactly like algebra, except \(i^2 = -1\)

- \(3(4 - 5i) = 12 - 15i\)
- \(2(3 + 2i) - 4(2 + i) = 6 + 4i - 8 - 4i = -2\)
- \(3i(1 + 2i) = 3i + 6i^2 = 3i - 6\) (Remember, \(i^2 = -1\))
- \((-3 + 2i)(4 - 5i)\)
  \[
  \begin{align*}
  &= (-3 + 2i)(4 - 5i) \\
  &= -12 + 15i + 8i - 10i^2 \\
  &= -12 + 23i + 10 \\
  &= -2 + 23i
  \end{align*}
  \]
Dividing Complex Numbers

1. Easy!
   - If it's just a real number on the bottom, divide each top term by the number on the bottom.

   \[ \frac{3 + 2i}{4 - 5i} \]
   - Example:
     - \[ \frac{10 + 15i}{5} = 2 + 3i \]
     - \[ \frac{23 - 17i}{4} = \frac{23}{4} - \frac{17}{4}i \]

2. Hard
   - If the bottom has an imaginary part, make it into the "easy" type.
   - How?
     - By multiplying top and bottom by the conjugate of the bottom.
     - Change the sign of the imaginary part.

Example:
- \[ \frac{5 + 5i}{1 + 2i} \]
  - **Step 1**: Multiply top and bottom by \((1 - 2i)\)
    - \[ \frac{(5 + 5i)(1 - 2i)}{(1 + 2i)(1 - 2i)} \]
  - **Step 2**: Lay it out like this
    - **Top**: \(5(1 - 2i) + 5i(1 - 2i) \Rightarrow 5 - 10i + 5i - 10i^2 \Rightarrow 5 - 5i + 10 \)
    - **Bottom**: \((1 + 2i)(1 - 2i) \Rightarrow 1 - 2i + 2i - 4i^2 \Rightarrow 1 + 4\)
  - **Steps 3 + 4**: Put \(\frac{\text{Top}}{\text{Bottom}}\) and divide
    - \[ \frac{15 - 5i}{5} = 3 - i \]
Using Complex Numbers for Quadratic Equations

Example:

\[ z^2 - 6z + 34 = 0 \]

\[ a = 1 \]
\[ b = -6 \]
\[ c = 34 \]

This can't be solved using real numbers alone.

\[ \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ \sqrt{(-6)^2 - 4(1)(34)} \]

\[ \sqrt{36 - 136} \]
\[ \sqrt{-100} \]
\[ \sqrt{100} \times \sqrt{-1} \]
\[ 10i \]

\[ \frac{6 \pm 10i}{2} \]

\[ \frac{3 + 5i}{2} \text{ or } \frac{3 - 5i}{2} \]

This will always happen. If the 2 answers are complex numbers, one will be the "conjugate" of the other.

You will need to practice lots of these.

\(1\) \[ z^2 - 10z + 29 = 0 \]
\(2\) \[ z^2 + 2z + 10 = 0 \]
\(3\) \[ z^2 - 12z + 37 = 0 \]
\(4\) \[ z^2 - 2z + 17 = 0 \]