DIFFERENTIATION / CALCULUS

THE RATE OF CHANGE OF ONE VARIABLE COMPARED TO ANOTHER.

> "THE MATHEMATICS OF CHANGE"

KEY WORDS:

- DERIVATIVE
- SLOPE OF TANGENT
- FIRST DERIVATIVE
- MAXIMUM / MINIMUM VALUES
- LIMIT.
- RATE OF CHANGE.

eg SPEED = HOW FAST YOUR DISTANCE IS CHANGING OVER TIME.

DIFFERENTIATION IS ALL ABOUT **SLOPES**

SLOPE = $\dfrac{\text{HOW FAST IS } y \text{ CHANGING}}{\text{HOW FAST IS } x \text{ CHANGING}}$ COMPARED TO $\dfrac{(\text{RISE})}{(\text{RUN})}$

↓

THIS IS WHAT DIFFO IS ALL ABOUT.

ANY QUESTION THAT MENTIONS:

- SLOPES
- MAXIMUM / MINIMUM VALUES
  METHOD $\left[\dfrac{dy}{dx} = 0\right]$
- RATE OF CHANGE

YOU NEED TO IMMEDIATELY THINK **DIFFO**

BASIC RULE OF DIFFO

IN WORDS : MULTIPLY THE NUMBER BY THE POWER AND REDUCE THE POWER BY 1

eg ①

$$y = 3x^5$$

$$\frac{dy}{dx} = 15x^4$$

②

$$y = 4x^2$$

$$\frac{dy}{dx} = 8x$$

③

$$y = 5x$$

$$\frac{dy}{dx} = 5$$

* NOTE WHEN YOU DIFF A CONSTANT, IT BECOMES ZERO

eg

$$y = 7$$

$$\frac{dy}{dx} = 0$$

NOTE THIS CAN BE DONE FOR MORE THAN ONE TERM

eg

$$y = x^5 + 4x^3 - 7x + 3$$

↓ DIFF ↓ DIFF ↓ DIFF ↓ DIFF

$$\boxed{\frac{dy}{dx} = 5x^4 + 12x^2 - 7}$$

2ND DERIVATIVE

SOMETIMES YOU NEED TO DIFFO TWICE [ONLY IF YOU'RE ASKED TO...]

$$y \xrightarrow{\text{DIFF}} \frac{dy}{dx} \xrightarrow{\text{DIFF}} \frac{d^2y}{dx^2}$$

OR

$$f(x) \rightarrow f'(x) \rightarrow f''(x)$$

OR

DISTANCE ↓ DIFFO SPEED ↓ ACCELERATION

EXTRA BIT

THEY MIGHT ASK YOU TO FIND $\frac{dy}{dx}$ AT A PARTICULAR POINT OR FOR A CERTAIN VALUE OF x.

eg IF $\boxed{y = x^2 - 5x}$, FIND THE VALUE OF $\frac{dy}{dx}$ WHEN $x = 1$

(COULD BE ANYTHING)

FIRST... FIND $\frac{dy}{dx} = 2x - 5$

THEN SUBSTITUTE. $\frac{dy}{dx}$ @ $x=1$ $= 2(1) - 5$

$= \boxed{-3}$ (USE YOUR CALCULATOR)

DON'T SUBSTITUTE INTO y. IT MUST BE $\frac{dy}{dx}$.

REMEMBER, 2 DIFFERENT NOTATIONS

$$y = f(x)$$

$$\frac{dy}{dx} = f'(x)$$

$$\frac{d^2y}{dx^2} = f''(x)$$

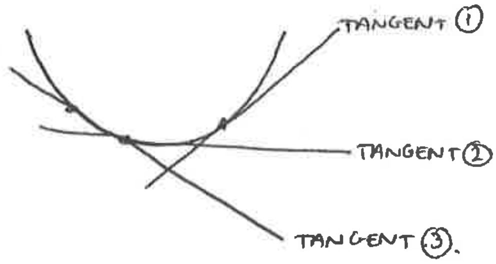
MAKE SURE YOU SUBSTITUTE INTO THE CORRECT ONE

ALL THIS TIME WHEN WE'VE BEEN JUST FOLLOWING SET ROUTINES OF HOW TO DIFFERENTIATE... WE HAVE BEEN WORKING OUT $\boxed{\text{SLOPES}}$

$\frac{dy}{dx}$ IS A ~~METHOD~~ FOR CALCULATING THE SLOPE OF THE TANGENT TO A CURVE AT ANY POINT OR FOR ANY VALUE OF x

eg. $y = x^2 - 2x + 3$

- IS A QUADRATIC FUNCTION.
- SO IT LOOKS LIKE THIS



- THE SLOPE OF THE TANGENT IS DIFFERENT / CHANGING FOR DIFFERENT VALUES OF x.

$$\boxed{\frac{dy}{dx} = 2x - 2}$$

(i) SLOPE at $x = 3$?

(ii) SLOPE at $(-1, 6)$?

(i) $\frac{dy}{dx}$ @ $x=3$ $= 2(3) - 2$ $= \boxed{4}$

(ii) $\frac{dy}{dx}$ @ $x=-1$ $= 2(-1) - 2$ $= \boxed{-4}$

IS IT INCREASING OR DECREASING ???...

IF $\frac{dy}{dx}$ IS:

POSITIVE $\longrightarrow$ INCREASING

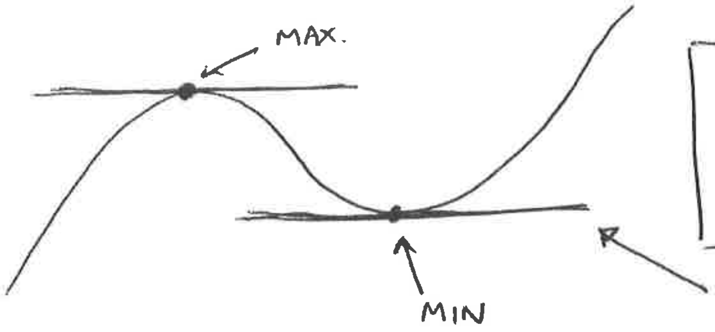
NEGATIVE $\longrightarrow$ DECREASING

DIFFICULT QUESTIONS

- THEY MIGHT ASK YOU TO FIND A POINT WHERE THE SLOPE IS A CERTAIN VALUE
- TO DO THIS... FIND $\frac{dy}{dx}$ AND MAKE IT = TO THE SLOPE THEY GAVE YOU.

1 TURNING POINTS MAXIMUM / MINIMUM

eg



REMEMBER :

$$\boxed{\text{MAX/MIN} \quad \frac{dy}{dx} = 0}$$

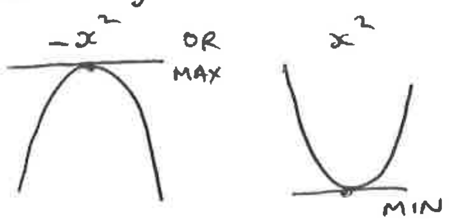
SLOPE = 0 (HORIZONTAL LINE)

- AT THE "TURNING POINTS", THE FUNCTION CHANGES FROM INCREASING TO DECREASING, OR VICE VERSA
- AT THESE POINTS, THE SLOPE OF THE TANGENT = 0

ie $\boxed{\frac{dy}{dx} = 0}$

QUADRATIC

(EASY) – 1 TURNING POINT



eg. $y = 2x^2 + 8x - 5$

$\frac{dy}{dx} = 4x + 8$

$\frac{dy}{dx} = 0$

SO $4x + 8 = 0$

$-8 \quad -8$

$4x = -8$

$\boxed{x = -2}$

NOW WE NEED TO FIND y.

$y = 2x^2 + 8x - 5$

$x=-2]$ $y = 2(-2)^2 + 8(-2) - 5 = -13$

SO $\boxed{(-2, -13)}$ IS THE MIN

CUBIC (HARD)

- 2 TURNING POINTS
- SOLVE QUADRATIC EQUATION.

eg $y = x^3 - 6x^2 + 9x - 10$

$\frac{dy}{dx} = 3x^2 - 12x + 9 = 0$

$\div 3$ $\quad x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x = 1 \qquad x = 3$

$\boxed{y = x^3 - 6x^2 + 9x - 10}$

$x=1$ $\qquad$ $x=3$

$y = (1)^3 - 6(1)^2 + 9(1) - 10$

$y = -6$

$(1, -6)$

MIN

$y = (3)^3 - 6(3)^2 + 9(3) - 10$

$y = 26$

$(3, 26)$

THIS IS MAX BECAUSE 26 IS HIGHER THAN -6.

RATES OF CHANGE / DISTANCE, SPEED, ACCELERATION

ANOTHER WAY TO THINK OF DIFFERENTIATION IS
HOW FAST IS ONE VARIABLE (USUALLY y)
CHANGING, COMPARED TO ANOTHER (USUALLY x)

SPEED = HOW FAST IS MY DISTANCE CHANGING OVER TIME?

ACCELERATION = HOW FAST IS MY SPEED CHANGING OVER TIME?

SO. IF I DIFF DISTANCE I GET SPEED
AND IF I DIFF SPEED I GET ACCELERATION

JUST REMEMBER THIS,

DISTANCE
↓ DIFF
SPEED
↓ DIFF
ACCELERATION

eg

$$\text{DISTANCE} = t^3 - 2t^2 + 3t$$ ← [THIS IS A FORMULA FOR DISTANCE]

↓ DIFF

$$\text{SPEED} = 3t^2 - 4t$$ ← [FORMULA FOR SPEED]

$$\text{ACCELERATION} = 6t$$ ← [FORMULA FOR ACCELERATION]

SO IF I WANT THE SPEED AFTER 3 SECONDS,
I USE THE SPEED FORMULA.

$$\text{SPEED} = 3t^2 - 4t$$
$$[t=3] = 3(3)^2 - 4(3)$$
$$= 27 - 12 = \boxed{15 \text{ m/s}}$$

IF A QUESTION IS USING HEIGHT FOR DISTANCE, THEY
OFTEN ASK ABOUT MAXIMUM HEIGHT (REMEMBER? MAX $\Rightarrow \frac{dy}{dx} = 0$)

AT THE HIGHEST POINT, $\boxed{\text{SPEED} = 0}$ → SOLVE.

LIMITS (LOOKS WEIRD – IT'S ACTUALLY VERY EASY!)

- WE SOMETIMES NEED TO KNOW THE "LIMIT" OF A FUNCTION FOR A PARTICULAR VALUE OF x.
- THIS SIMPLY MEANS: WHAT IS THE VALUE OF THE FUNCTION AT THIS POINT?
- THIS USUALLY MEANS SUBSTITUTE THE VALUE OF x INTO THE FUNCTION.

eg $$\lim_{x \to 3} (x+4)$$

SUBTITUTE 3 IN FOR x

$$= 3+4$$

$$= 7$$

THIS MEANS / WE SAY: "WHAT IS THE "LIMIT" OF $x+4$ WHERE $x=3$"

- IT CAN BE MORE DIFFICULT IF THE FUNCTION IS "UNDEFINED" AT THAT POINT. (ie. IF SUBSTITUTION DOESN'T WORK)

eg $$\lim_{x \to 4} \frac{x^2-16}{x-4}$$

IF SUBSTITUTION DOESN'T WORK $\Rightarrow$ TIDY UP FIRST

- IF WE SUBSTITUTE $x=4$ INTO THE BOTTOM, WE WOULD BE DIVIDING BY $4-4$ WHICH IS 0. → WHICH IS NOT ALLOWED!!
- INSTEAD, FACTORISE + TIDY UP FIRST

ie x^2-16 IS $(x-4)(x+4)$ [D.O.T.S]

SO $\dfrac{\cancel{(x-4)}(x+4)}{\cancel{(x-4)}} = x+4$ AND $\lim_{x \to 4} x+4 = \boxed{8}$