FUNCTIONS

A FUNCTION IS LIKE A "RULE", OR A "MACHINE". FOR EVERY INPUT, A FUNCTION GIVES US A PARTICULAR OUTPUT.

WAYS OF SHOWING A FUNCTION:

MAPPING DIAGRAM

TABLE

FUNCTION NOTATION

\[
\begin{align*}
\text{INPUT} & \quad \text{FUNCTION (RULE)} & \quad \text{OUTPUT} \\
-1 & \quad \text{ADD 3} & \quad 2 \\
0 & \quad \text{ADD 3} & \quad 3 \\
1 & \quad \text{ADD 3} & \quad 4 \\
2 & \quad \text{ADD 3} & \quad 5 \\
\end{align*}
\]

\[
\begin{align*}
f(x) &= \frac{x^2}{x} \\
f(1) &= (1)^2 = 1 \\
f(2) &= (2)^2 = 4 \\
f(7) &= (7)^2 = 49 \\
\end{align*}
\]

DOMAIN: ALL OF THE INPUTS

RANGE: ALL OF THE OUTPUTS

COUPLES/PAIRS

\[
\left\{ (1, 2), (2, 4), (3, 6) \right\}
\]

INPUT: \( x \)

OUTPUT: \( y \)
**Important Notes / Terms**

- \( x \) = **Input**
- \( y \) = **Output** \( \rightarrow \) **Also** \( y = f(x) \)

**Domain** = **All The Inputs**

**Range** = **All The Outputs**

**Codomain** = **All The (Possible) Outputs**

**Two Inputs Could Have The Same Output.**

\( \text{eg} \quad f(x) = x^2 \)

\( f(1) = (1)^2 = 1 \)

\( \text{and} \quad f(-1) = (-1)^2 = 1 \)

**Different Inputs** \( \rightarrow \) **Same Outputs**

**But**

**One Input Can't Have Two Different Outputs.**

i.e. **There Is Only One Output For Each Input.**

---

**Substitution:**

- If you're given the function \( \text{eg} \quad f(x) = 2x + 3 \)
- Replace the \( x \) with each input (in a bracket)
- So \( f(2) = 2(2) + 3 \)
- \( f(2) = 7 \)
- \((2, 7)\)
**Composite Functions**

WHERE WE HAVE TO DO ONE FUNCTION AFTER ANOTHER...

\[ \text{eg } f(x) = 3x + 4 \quad \text{AND} \quad g(x) = x^2 \]

**Find**

(i) \( f \circ g(2) \)

(ii) \( g(f(2)) \)

(i) \( f \circ g(2) \) \( \leftarrow \) MEANS \( f \) AFTER \( g(2) \)

\[ g(2) = (2)^2 = 4 \]

\[ f(4) = 3(4) + 4 = 16 \]

(ii) \( g(f(2)) \) \( \leftarrow \) MEANS \( g \) OF \( f(2) \)

\[ f(2) = 3(2) + 4 = 10 \]

\[ g(f(2)) = g(10) = (10)^2 = 100 \]

**IN GENERAL** **THE ORDER MATTERS**

\( g \circ f(x) \) IS **NOT** THE SAME AS \( f \circ g(x) \)
FUNCTIONS CAN BE GRAPHED ON THE X-AXIS AND Y-AXIS.

REMEMBER THAT A FUNCTION IS A RULE THAT "MAPS" A PARTICULAR INPUT TO A PARTICULAR OUTPUT.

THESE CAN BE WRITTEN AS PAIRS OF INPUTS + OUTPUTS: \((x, y)\) OR \((x, f(x))\).

THESE POINTS CAN THEN BE PLOTTED ON X-AXIS / Y-AXIS, MAKING A CERTAIN SHAPE.

WE NEED TO BE FAMILIAR WITH 4 PARTICULAR TYPES OF FUNCTION:

1. **LINEAR**
   \(y = 3x + 2\)
   \(f(x) = 3x + 2\)
   \(f(x) = 3x^2 + 2\)

2. **QUADRATIC**
   \(f(x) = x^2 + 3x - 4\)
   \(f(x) = x^2 + 3x - 4\)

3. **CUBIC**
   \(f(x) = x^3 - 3x^2 + 2x + 4\)
   \(f(x) = x^3 - 3x^2 + 2x + 4\)

4. **EXPONENTIAL**
   \(f(x) = 2^{3^x}\)
   \(f(x) = 2^{3^x}\)
Transformations

[What happens when we change the original function?]

[Use the graphing applets on the website]

Linear - [Easy] - We know from coordinate geometry that

\[ y = mx + c \]

So in our functions, if we change

[eg \( f(x) = 3x + 2 \)]

The numbers, we are altering the slope or the y-intercept.

* Remember, parallel lines have the same slope

If we change the "m" part, the slope changes

If we play around with the "+c" part, the graph moves up or down

Quadratics \( y = ax^2 + bx + c \)

- If we change \( a \), the graph gets steeper/narrower
- If we change \( c \), the graph moves up/down (don't worry about \( b \))

\[ y = (x + b)^2 \]

- Changing \( b \) moves the graph left/right
CUBICS \( y = ax^3 + bx^2 + cx + d \)

- Changing \( d \) moves graph \( \uparrow \) or \( \downarrow \)
- Changing \( a \) makes it tall + skinny or short + fat.
  (Don't worry about \( b \) and \( c \))

\( y = (x+b)^3 \)

→ As with quadratics, \( b \) moves the graph \( \leftarrow \) or \( \rightarrow \)

EXPONENTIAL \( y = a k^x + b \)

→ Changing \( a \) makes it steeper
→ Changing \( b \) moves it \( \uparrow \) or \( \downarrow \)
→ Changing \( x \) [the power] moves graph \( \leftarrow \) or \( \rightarrow \)

For all these transformations, try making a table + plotting new graphs to get an idea of the new shape.