INDICES

THIS IS ANOTHER WORD FOR "POWERS"

eg 2^6

$2 \times 2 \times 2 \times 2 \times 2 \times 2$

- WHAT DO INDICES MEAN ?

2^6 ← POWER

BASE →

THE "BASE" MULTIPLIED BY ITSELF THE NUMBER OF TIMES INDICATED BY THE "POWER"

THIS MEANS → $2 \times 2 \times 2 \times 2 \times 2 \times 2$

TWO MULTIPLIED BY ITSELF 6 TIMES.

- THIS IS NOT THE SAME AS THIS

2^6 ≠ 2×6.

DO NOT MAKE THIS MISTAKE

eg $3^4 = 3 \times 3 \times 3 \times 3$

$5^3 = 5 \times 5 \times 5$

$(-4)^2 = (-4) \times (-4)$

$a^5 = a \times a \times a \times a \times a$

MULTIPLYING POWERS / INDICES

eg $5^3 \times 5^4$?

THIS MEANS $(5 \times 5 \times 5)$

THIS MEANS $(5 \times 5 \times 5 \times 5)$

WHICH MEANS / IS THE SAME AS

$5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^7$

SO, $5^3 \times 5^4 = 5^7$

IF BASE IS THE SAME

MULTIPLYING

① RULE

ADD THE "POWERS"

eg $a^2 \times a^3 = a^5$

DIVIDING POWERS / INDICES

eg $\frac{6^5}{6^3} = \frac{6 \times 6 \times 6 \times 6 \times 6}{6 \times 6 \times 6}$ ← NOW WE CAN "CANCEL" THE 6's ON TOP AND BOTTOM.

$\frac{\cancel{6} \times \cancel{6} \times \cancel{6} \times 6 \times 6}{\cancel{6} \times \cancel{6} \times \cancel{6}} = 6^2$

SO $\frac{6^5}{6^3} = 6^2$

② DIVIDING / FRACTION RULE

SUBTRACT THE POWERS.

YOU NEED TO KNOW THESE RULES, AND EQUALLY IMPORTANTLY, YOU NEED TO KNOW WHY THEY WORK.

A POWER TO A POWER (BRACKETS)

eg $(2^3)^4$

MEANS $(2\times2\times2)^4$, SO $(2\times2\times2)\times(2\times2\times2)\times(2\times2\times2)\times(2\times2\times2)$

$(2^3)^4 = 2^{12}$

(3) **RULE**
MULTIPLY THE POWERS

$(\text{ANYTHING})^0 = 1$ (4)

YOU NEED TO JUST REMEMBER THIS ONE. eg.

$5^0 = 1$; $3^0 = 1$; $2^0 = 1$

WHY ?

eg $\frac{3^7}{3^7}$ REMEMBER RULE ② ? (SUBTRACT THE POWERS)

So $\frac{3^7}{3^7} = 3^{7-7} = 3^0$

AND WE ALSO KNOW, $\frac{\text{ANYTHING}}{\text{ITSELF}} = 1$

eg $\frac{5}{5} = 1$; $\frac{2}{2} = 1$; $\frac{-4}{-4} = 1$

$\frac{3^7}{3^7} = 3^0$

$\frac{3^7}{3^7} = 1$

→ SO $3^0 = 1$

NEGATIVE POWERS

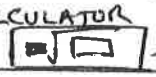
eg ⑤ $\boxed{3^{-4} = \frac{1}{3^4}}$

YOU NEED TO KNOW THIS BACKWARDS.

SO $\frac{1}{a^5} = a^{-5}$

FRACTIONAL POWERS = ROOTS

$\boxed{a^{\frac{1}{k}} = \sqrt[k]{a}}$ " k^{TH} ROOT "

[BUTTON ON CALCULATOR ]

eg $\boxed{16^{\frac{1}{2}} = \sqrt{16}}$ ⑥

$a^{1/2} = \sqrt{a}$

THERE ARE OTHER LAWS OF INDICES, BUT THESE ARE THE MAIN ONES ....

THE LAWS / RULES ARE ON p 21 OF THE TABLES BOOK. (BUT IT IS SLIGHTLY CONFUSING AND BADLY WRITTEN)

THE MOST IMPORTANT THING IS TO UNDERSTAND WHAT INDICES / POWERS MEAN AND BE ABLE TO WORK OUT QUESTIONS YOURSELF

eg $3^4 \times 3^3$ IF I FORGET THE RULE, THIS MEANS

$$\frac{3\times3\times3\times3}{3^4} \times \frac{3\times3\times3}{3^3}$$

WHICH IS $3\times3\times3\times3\times3\times3\times3$

$= 3^7$

EQUATIONS WITH INDICES

- THESE ARE TRICKY, BUT NOT IMPOSSIBLE.
- MAIN RULE:
  - GET SAME "BASE" ON BOTH SIDES
  - LET POWER = POWER

eg $2^x = 2^5$ ← SAME BASE

SO $x = 5$

- THEY ARE USUALLY NOT THIS EASY...

eg $2^x = 32$ ← PROBLEM: NOT SAME BASE

REWRITE AS $32 = 2^5$

$2^x = 2^5$ ← SAME BASE!

NOW $x = 5$

- HARD EXAMPLE

$$3^{3x-1} = \left(\frac{27}{\sqrt{3}}\right)^5$$

$27 = 3^3$ $\quad \sqrt{3} = 3^{1/2}$

UGH... WE NEED THIS AS $3^{\text{SOMETHING}}$

SO $\dfrac{27}{\sqrt{3}} = \dfrac{3^3}{3^{1/2}}$

WHICH EQUALS $3^{2\frac{1}{2}}$ [SUBTRACT POWERS]

NOW, $3^{3x-1} = \left(3^{2\frac{1}{2}}\right)^5$

[BRACKETS => MULTIPLY POWERS]

$5 \times 2\frac{1}{2} = \frac{25}{2}$

$3^{3x-1} = 3^{\frac{25}{2}}$

SO $3x - 1 = \frac{25}{2}$ $\xrightarrow{\times 2}$ $6x - 2 = 25$ $\xrightarrow{+2}$

$6x = 27$

$x = 4\frac{1}{2}$

SURDS ($\sqrt{\ }$) – SQUARE ROOTS WHICH DON'T HAVE A NICE ANSWER.

→ THESE ARE IRRATIONAL NUMBERS

eg $\sqrt{2}$ or $\sqrt{3}$

* MOST IMPORTANT THING

WHEN YOU "SQUARE" A SURD IT CANCELS OUT. (BECAUSE THEY ARE OPPOSITES OF EACH OTHER)

eg $\sqrt{9} = 3$

$3^2 = 9$ ⟶ SO $(\sqrt{9})^2 = 9$

AND $(\sqrt{7})^2 = 7$

AND ALSO $\sqrt{6^2} = 6$

OTHER STUFF TO KNOW

- $\sqrt{3} \times \sqrt{2} = \sqrt{6}$ [YOU CAN MULTIPLY]
- $\frac{\sqrt{9}}{\sqrt{3}} = \sqrt{\frac{9}{3}} = \sqrt{3}$ [YOU CAN DIVIDE]

LET YOUR CALCULATOR DO MOST OF THESE QUESTIONS FOR YOU.

- IF YOU HAVE AN EQUATION WITH SURDS, SQUARE BOTH SIDES TO GET RID OF THE SURDS.
- IF A QUESTION SAYS 'IN SURD FORM' IT MEANS IT HAS A $\sqrt{\ }$ SIGN IN IT.