INDICES

**WHAT DO INDICES MEAN?**

- **Base** \( 2 \) → **Power** \( 6 \)

- The "base" multiplied by itself **the number of times indicated by the power.**

- **This means** \( 2 \times 2 \times 2 \times 2 \times 2 \times 2 \)

- Two multiplied by itself **6 times.**

**THIS IS NOT THE SAME AS THIS**

\[ \quad 2^6 \]

**DO NOT MAKE THIS MISTAKE**

\[ \quad 2 \times 6 \]

\[ \text{Eq } 3^4 = 3 \times 3 \times 3 \times 3 \]

\[ 5^3 = 5 \times 5 \times 5 \]

\[ (-4)^2 = (-4) \times (-4) \]

\[ a^5 = a \times a \times a \times a \times a \]
**MULTIPLYING**

**POWERS / INDICES**

\[ 5^3 \times 5^4 \]

**Rule:**

1. **ADD THE **"**POWERS"**

\[ a^2 \times a^3 = a^5 \]

\[ 1 \]

**This means:**

\[ (5 \times 5 \times 5) \times (5 \times 5 \times 5 \times 5) \]

**Which means:**

\[ 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \]

**So:**

\[ 5^7 \]

**IF BASE IS THE SAME**

**DIVIDING**

**POWERS / INDICES**

\[ \frac{6^5}{6^3} \]

**Rule:**

2. **SUBTRACT THE POWERS.**

\[ a^5 \div a^3 = a^2 \]

\[ \frac{6 \times 6 \times 6 \times 6 \times 6}{6 \times 6 \times 6} \]

\[ \frac{6 \times 6 \times 6 \times 6}{6 \times 6 \times 6} \]

**So:**

\[ 6^2 \]

**NOW WE CAN "CANCEL" THE 6's ON TOP AND BOTTOM.**

**YOU NEED TO KNOW THESE RULES, AND EQUALLY IMPORTANTLY, YOU NEED TO KNOW WHY THEY WORK.**
A POWER TO A POWER (BRACKETS)

\[ \text{eq} \quad (2^3)^4 \]

**Means**
\[
(2 \times 2 \times 2)^4, \quad \text{so} \quad (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \]

\[
(2^3)^4 = 2^{12} \quad \text{(3)} \]

**Rule**
**Multiply the powers**

\[ \text{(Anything)}^0 = 1 \]

\[ \text{You need to just remember this one. eq.} \]

\[ 5^0 = 1; \quad 3^0 = 1; \quad 2^0 = 1 \]

**Why?**

\[ \text{eq} \quad \frac{3^7}{3^7} \]

**Remember Rule 3?**

\[ \text{(Subtract the powers)} \]

\[ \frac{3^7}{3^7} = 3^{7-7} = 3^0 \]

\[ \text{And we also know, Anything} \frac{\text{anything}}{\text{itself}} = 1 \]

\[ \text{eq} \quad \frac{5^1}{5^1} = 1; \quad \frac{2^2}{2^2} = 1; \quad \frac{-4}{-4} = 1 \]

\[ \text{so} \quad 3^0 = 1 \]
NEGATIVE POWERS

\[ 3^{-4} = \frac{1}{3^4} \]

You need to know this backwards.

So

\[ \frac{1}{a^5} = a^{-5} \]

FRACTIONAL POWERS = ROOTS

\[ a^{\frac{1}{k}} = \sqrt[k]{a} \]

\[ \sqrt[k]{a} = a^{\frac{1}{k}} \]

There are other laws of indices, but these are the main ones....

The laws/rules are on p 21 of the tables book. (It is slightly confusing and badly written)

The most important thing is to understand what indices/powers mean and be able to work out questions yourself.

\[ 3^4 \times 3^3 \]

If I forget the rule, this means

\[ \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 }{3^6} \times 3^3 \]

Which is

\[ 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \]

= 3^7
EQUATIONS WITH INDICES

* These are tricky, but not impossible.

**Main Rule:**
- Get same "base" on both sides
- Let power = power

\[ \text{Eq} \quad 2^x = 2^5 \quad \rightarrow \quad \text{same base} \]

So \( x = 5 \)

* They are usually not this easy...

\[ \text{Eq} \quad 2^x = 32 \quad \rightarrow \quad \text{not same base} \]

Rewrite as \( 2^x = 2^5 \quad \rightarrow \quad \text{same base}! \)

Now \( x = 5 \)

* Hard example

\[ 3^{3x-1} = \left( \frac{27}{\sqrt{3}} \right)^5 \]

\[ 27 = 3^3 \quad \sqrt{3} = 3^{1/2} \]

So \[ \frac{27}{\sqrt{3}} = \frac{3^3}{3^{1/2}} \]

Which equals \[ 3^{2\frac{1}{2}} \quad \text{[subtract powers]} \]

\[ \left( 3^{2\frac{1}{2}} \right)^5 \]

\[ 5 \times 2\frac{1}{2} = 25 \]

\[ 3^{3x-1} = 3^{2\frac{1}{2}} \]

So \[ 3x - 1 = \frac{25}{2} \quad \Rightarrow \]

\[ 6x - 2 = 25 \]

\[ 6x = \frac{27}{2} \quad \Rightarrow \]

\[ 6x = 27 \quad \Rightarrow \]

\[ x = \frac{27}{6} \quad \Rightarrow \]

\[ x = \frac{9}{2} \]
SURDS (\(\sqrt{\cdot}\)) - SQUARE ROOTS WHICH DON'T HAVE A NICE ANSWER.

\[\Rightarrow\] THESE ARE IRRATIONAL NUMBERS

e.g. \(\sqrt{2}\) or \(\sqrt{3}\)

* MOST IMPORTANT THING

WHEN YOU "SQUARE" A SURD
IT CANCELS OUT. (BECAUSE THEY ARE OPPOSITES OF EACH OTHER)

\[\begin{align*}
\text{e.g.} & \quad \sqrt{9} = 3 \\
& \quad 3^2 = 9 \quad \rightarrow \quad \text{SO} \quad (\sqrt{9})^2 = 9 \\
& \quad \text{AND} \quad (\sqrt{7})^2 = 7 \\
& \quad \text{AND ALSO} \quad \sqrt{6^2} = 6
\end{align*}\]

OTHER STUFF TO KNOW

\[\begin{align*}
& \cdot \sqrt{3} \times \sqrt{2} = \sqrt{6} \quad [\text{YOU CAN MULTIPLY}] \\
& \cdot \frac{\sqrt{9}}{\sqrt{3}} = \frac{\sqrt{9}}{\sqrt{3}} \times \sqrt{3} \quad [\text{YOU CAN DIVIDE}]
\end{align*}\]

LET YOUR CALCULATOR DO MOST OF THESE QUESTIONS FOR YOU.

- IF YOU HAVE AN EQUATION WITH SURDS,
  SQUARE BOTH SIDES TO GET RID OF THE SURDS.

- IF A QUESTION SAYS "IN SURD FORM" IT MEANS IT HAS A \(\sqrt{\cdot}\) SIGN IN IT.