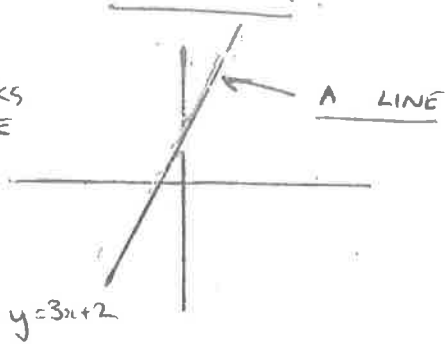
CO-ORDINATE GEOMETRY - [THE LINE]

- CO-ORDINATE GEOMETRY IS <u>VERY</u> IMPORTANT.
- IT LINKS <u>ALGEBRA</u> AND <u>GEOMETRY</u>
- SO $\downarrow$

$$y = 3x + 2$$

LOOKS LIKE



EVERY POINT ON THE CO-ORDINATE DIAGRAM HAS

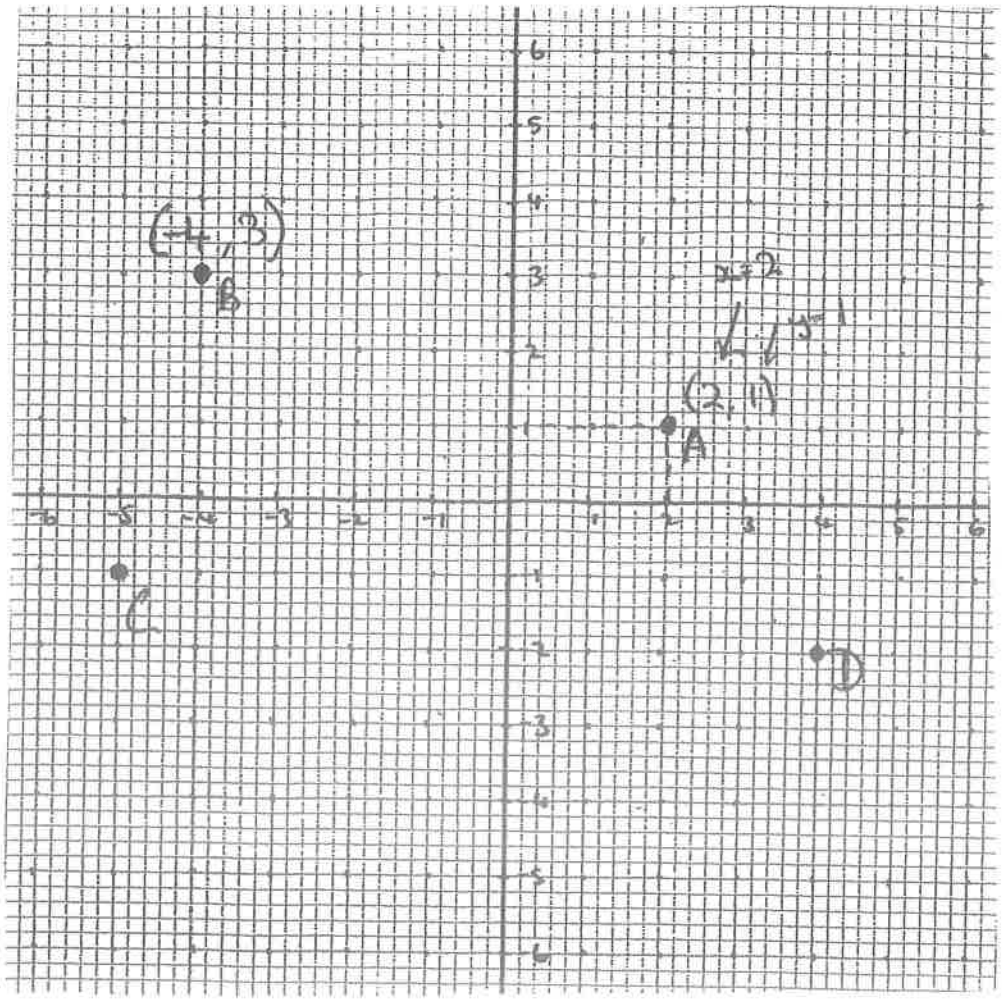
X-CO-ORDINATE Y-CO-ORDINATE

(x, y)

THINK OF YOUR ALPHABET. X COMES BEFORE Y.

$\uparrow$

DON'T MIX THESE UP



A = (2, 1)

B = (-4, 3)

C = (-5, -1)

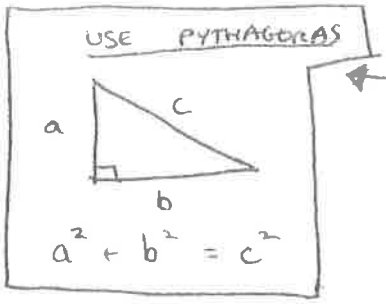
D = (4, -2)

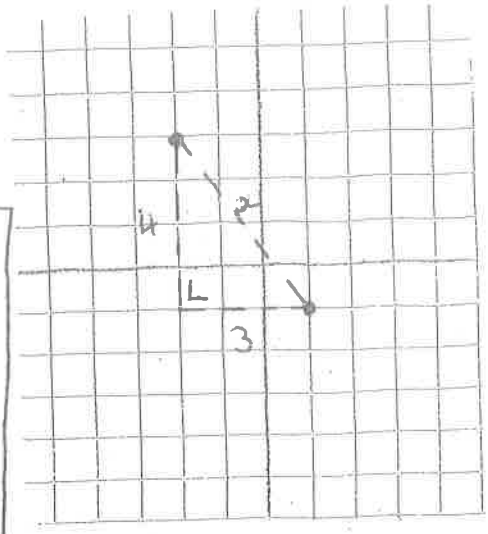
DISTANCE (HOW FAR?)

- YOU MIGHT BE ASKED TO FIND THE DISTANCE BETWEEN 2 POINTS.

- THE EASIEST WAY IS TO

  - PLOT THE TWO POINTS
  - MAKE A RIGHT-ANGLED TRIANGLE
  - FIND THE LENGTH OF THE HYPOTENUSE





eg FIND THE DISTANCE BETWEEN $(-2, 3)$ AND $(1, -1)$

$3^2 + 4^2 = x^2$

$9 + 16 = x^2$

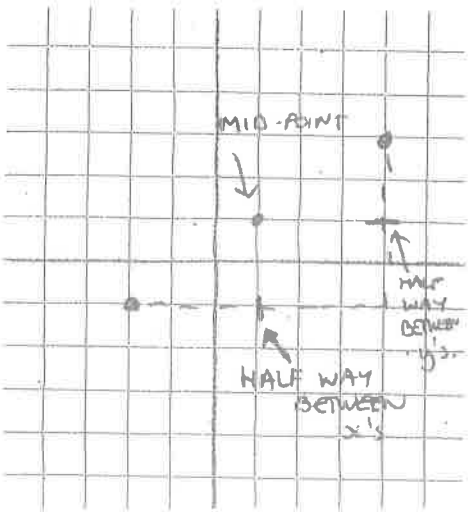
$25 = x^2$

$x = 5$

- THE OLD-FASHIONED WAY IS TO USE THE FORMULA $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

MID-POINT (HALF-WAY POINT)

- HALF-WAY BETWEEN THE x's AND HALF-WAY BETWEEN THE y's.
- DRAW A DIAGRAM. (USE SQUARED PAPER)
- WHAT IS HALF-WAY BETWEEN TWO NUMBERS?
  THE AVERAGE
  ADD TOGETHER + DIVIDE BY 2



eg WHAT IS THE MID-POINT OF $(-2, -1)$ AND $(4, 3)$

OR USE THE OLD-FASHIONED FORMULA

$$\left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right)$$

SLOPE (HOW STEEP IS THAT LINE?)

WE USE THE LETTER **m** FOR SLOPE

eg $m = 1$

LESS STEEP, SMALLER SLOPE

eg $m = \frac{1}{2}$

FLAT, NO SLOPE

eg $m = 0$

DOWNHILL $m = -1$

eg

MORE STEEP, BIGGER NUMBER

eg $m = -2$

eg $m = 2$

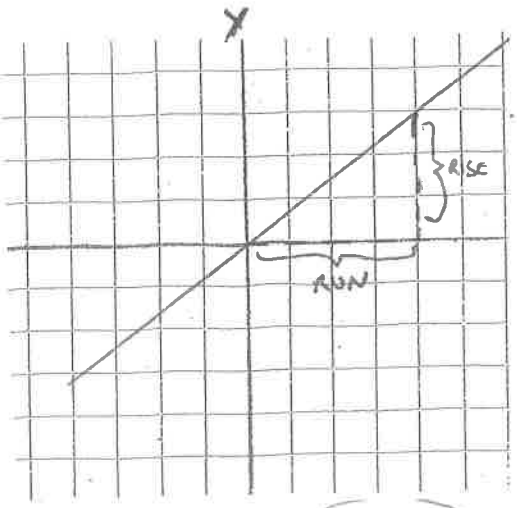
+ SLOPE = INCREASING

− SLOPE = DECREASING

SLOPE = $\frac{\text{RISE}}{\text{RUN}}$ → HOW HIGH DOES IT GO UP COMPARED TO HOW FAR IT GOES ACROSS

REMEMBER THAT'S WHAT SLOPE IS

eg



$\frac{\text{RISE}}{\text{RUN}} = \frac{3}{4}$

IF YOU'RE ASKED TO FIND SLOPE, DRAW X+Y AXES; PLOT POINTS; WORK OUT $\frac{\text{RISE}}{\text{RUN}}$

OR

OLD FASHIONED WAY:

eg $(2, 3)$ $(5, 4)$

x_1 y_1 x_2 y_2

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 3}{5 - 2} = \boxed{\frac{1}{3}}$$

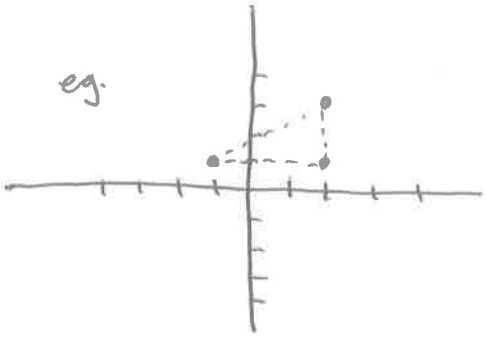
① LABEL POINTS (x_1, y_1) AND (x_2, y_2)

② USE SLOPE FORMULA $m = \frac{y_2 - y_1}{x_2 - x_1}$

SLOPE / MID-POINT / DISTANCE

- DRAW X + Y AXES
- PLOT 2 POINTS
- DRAW OUT THE RIGHT-ANGLED TRIANGLE
- WORK OUT THE LENGTHS OF THE SIDES

eg.



BE VERY CAREFUL ABOUT WHICH THING YOU ARE LOOKING FOR...

DON'T GET THE SLOPE WHEN YOU WERE ASKED FOR DISTANCE

- MAKE SURE YOU ANSWER THE EXACT QUESTION THEY ASKED...

DOUBLE-CHECK THIS AT THE END!!

EQUATION OF A LINE

- EVERY LINE THAT YOU CAN DRAW ON THE X+Y AXES HAS A MATHEMATICAL "NAME", OR "EQUATION"

eg $3x + 2y = 5$ OR $y = 2x + 1$

2 DIFFERENT WAYS OF WRITING THE "EQUATION OF A LINE"

- WHAT IS THE DIFFERENCE BETWEEN ANY 2 LINES?
  - HOW SLANTED ARE THEY? / OR —
  - WHERE ARE THEY

SLOPE

POSITION

2 IMPORTANT THINGS TO DISTINGUISH BETWEEN LINES

2 FORMULAS

OR

$$y - y_1 = m(x - x_1)$$

USE THIS ONE WHEN WE KNOW A POINT ON THE LINE

eg $m = 2$ $(x_1, y_1) = (3, 4)$

$y - 4 = m(x - 3)$

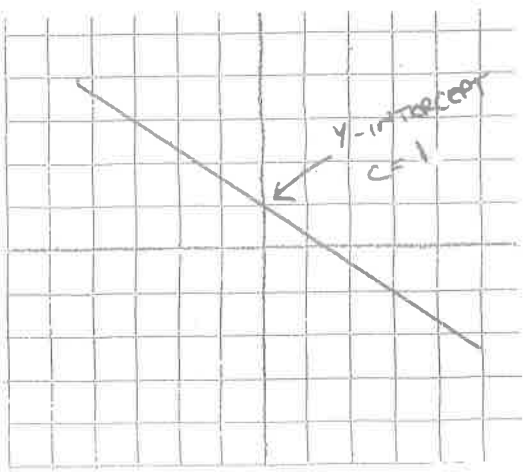
NOTE DON'T REPLACE THE y OR x IN THE FORMULA!

Y-INTERCEPT

$$y = mx + c$$

USE THIS ONE WHEN YOU KNOW WHERE THE LINE CROSSES THE Y-AXIS

THIS IS c



EQUATION OF A LINE CONTD...

- BE ABLE TO LOOK AT A LINE AND WRITE EQUATION OF THE LINE BY LOOKING AT
  - SLOPE (m)
  - Y-INTERCEPT (c)

USE $y = mx + c$

- BE ABLE TO WRITE DOWN THE SLOPE OF A LINE VERY QUICKLY

eg $y = 3x + 9$

SLOPE = 3

BEFORE DOING THIS, YOUR EQUATION OF A LINE HAS TO LOOK LIKE THIS

$y = mx + c$

- ANY 2 PARALLEL LINES HAVE THE SAME SLOPE

So $y = 2x + 4$ IS PARALLEL TO $y = 2x - 1$

- PERPENDICULAR LINES = 90° ANGLES

THESE ARE MORE DIFFICULT.

IF LINES ARE PARALLEL, WHEN YOU MULTIPLY THE SLOPES, YOU GET -1.

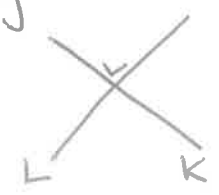
YOU MIGHT BE ASKED TO PROVE THAT 2 LINES ARE PERPENDICULAR. MULTIPLY THE SLOPES.

$\text{SLOPE}_1 \times \text{SLOPE}_2 = -1$

TRICK

CHANGE THE SIGN AND TURN IT UPSIDE-DOWN

eg SLOPE OF L IS 2



CALCULATE THE SLOPE OF K

$m = -\frac{1}{2}$

CHANGE SIGN + TURN IT UPSIDE-DOWN

eg $2 \times -\frac{1}{2} = -1$

PERPENDICULAR

GRAPHING LINES

- TO GRAPH A LINE, WE NEED 2 POINTS
- THE EASIEST POINTS TO FIND ARE WHERE $x = 0$, $y = 0$
- USE THE "T" METHOD

eg GRAPH $2x + 3y = 6$

② $2x + 3y = 6$	
③ $x = 0$	③ $y = 0$
④ $2(0) + 3y = 6$ $3y = 6$ $y = 2$	⑤ $2x + 3(0) = 6$ $2x = 6$ $x = 3$
⑥ $(0, 2)$	$(3, 0)$

REMEMBER (x, y)

① DRAW THE BIG T

② WRITE THE EQUATION OF THE LINE ON TOP

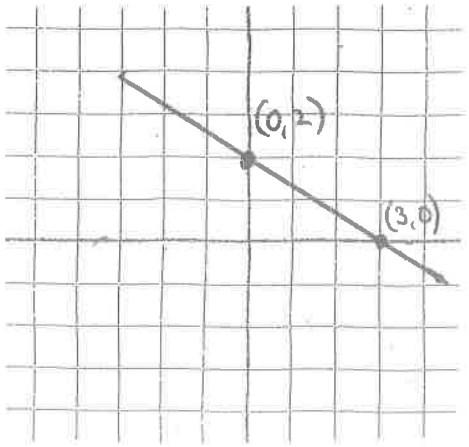
③ WRITE DOWN $x = 0$ AND $y = 0$

④ SUBSTITUTE THESE VALUES OF 0 INTO THE EQUATION

⑤ SOLVE FOR THE OTHER LETTER

⑥ WRITE DOWN YOUR TWO POINTS

⑦ PLOT THESE POINTS + LINK WITH RULER.

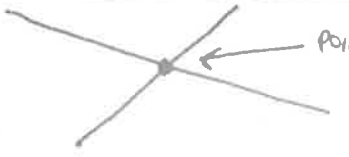


PRACTICE A FEW OF THESE

NOTE
- YOU MAY BE ASKED TO GRAPH 2 LINES ON THE SAME DIAGRAM
- DO EACH ONE USING THE METHOD IN THE BOX ABOVE

POINT OF INTERSECTION : THIS IS THE POINT WHERE 2 LINES CROSS (INTERSECT)

eg



AREA OF A TRIANGLE

FORMULA : (GIVEN IN TABLES BOOK)

$$\frac{1}{2}\left| x_1 y_2 - x_2 y_1 \right|$$

THIS ONLY WORKS WHEN ONE OF THE POINTS IS $(0,0)$

↑ SOMETIMES WE HAVE TO MAKE THIS HAPPEN FIRST.

THE | SQUARE BRACKETS | JUST MEAN THAT IF YOU GET A NEGATIVE ANSWER, YOU MAKE IT INTO A PLUS.

HOW DOES ? THIS WORK.

EXAMPLE

FIND THE AREA OF A TRIANGLE FORMED BY

(a) $(0,0)$ $(2,1)$ $(-1,3)$

GREAT

x_1 y_1 x_2 y_2

FORMULA : $\frac{1}{2}\left| x_1 y_2 - x_2 y_1 \right|$

BE CAREFUL WITH SIGNS →

$= \frac{1}{2}\left|(2)(3) - (-1)(1)\right|$

$= \frac{1}{2}\left|6 + 1\right|$

$= \frac{1}{2}\left|7\right|$

$= \boxed{3.5}$

(b) $(2,3)$ $(0,4)$ $(-2,7)$

HARD →

NONE OF THESE IS $(0,0)$

WE'LL PICK ONE OF THEM AND CHANGE IT INTO $(0,0)$

SO,

$(2,3)$ $(0,4)$ $(-2,7)$

-4

$(0,0)$

DO THIS TO THE OTHER 2 POINTS

-4

$(2,-1)$ $(-2,3)$

x_1 y_1 x_2 y_2

$\frac{1}{2}\left| x_1 y_2 - x_2 y_1 \right|$

$= \frac{1}{2}\left|(2)(3) - (-2)(-1)\right|$

$= \frac{1}{2}\left|6 - 2\right|$

$= \frac{1}{2}\left|4\right|$

$= \boxed{2}$