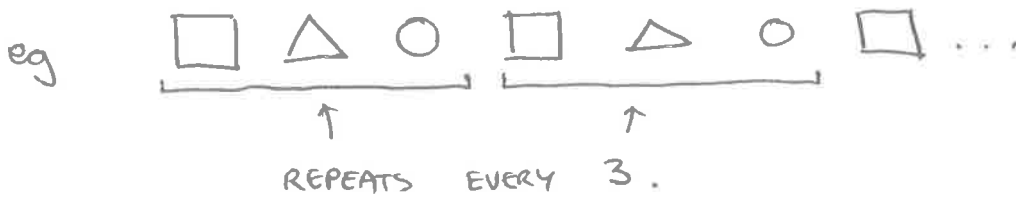


PATTERNS + SEQUENCES

- WHAT MATHS IS ALL ABOUT !!
- WE WANT TO TAKE A LIST / PATTERN AND MAKE GENERAL MATHEMATICAL FORMULAS TO DESCRIBE IT OR TO MAKE GENERALISATIONS ABOUT THE PATTERN.

REPEATING PATTERNS

COLOURS
SHAPES
BLOCKS



WHAT SHAPE WILL 9TH BE ?

○ - WHY? - BECAUSE EVERY 3RD SHAPE IS A ○

WHAT SHAPE WILL 101ST SHAPE / BLOCK BE ?

$$101 \div 3 = 33 \text{ REMAINDER } 2.$$

SO THE 99TH BLOCK WILL BE A ○
(33 × 3)

$$100^{\text{TH}} = \square, \quad 101^{\text{ST}} = \triangle.$$

TO MAKE SENSE OF A NUMBER PATTERN.

- WRITE DOWN A FEW TERMS, TRY TO WORK OUT WHAT THE NEXT 2 WILL BE...

- MAKE A TABLE → USE THIS TO SEE WHAT'S HAPPENING.

TYPES OF SEQUENCE / PATTERNS YOU WILL SEE :

① LINEAR (ALSO KNOWN AS ARITHMETIC)

- MOST IMPORTANT

- GOES UP BY SAME AMOUNT EACH TIME.
(1st DIFFERENCE THE SAME)

eg: 3, 7, 11, 15
GOES UP IN 4'S

② QUADRATIC

- THE AMOUNT IT GOES UP BY CHANGES — NOT LINEAR.
- THE AMOUNT IT GOES UP BY GOES UP BY THE SAME AMOUNT EACH TIME
(2nd DIFFERENCE THE SAME)

eg: 2 5 10 17

 ┌───┐ ┌───┐ ┌───┐ ← 1st DIFFERENCE

 +3 +5 +7

 ┌───┐ ┌───┐ ← 2nd DIFFERENCE = 2

 +2 +2

③ EXPONENTIAL (DOUBLING / TRIPLING)

eg 3, 6, 12, 24 DOUBLING EACH TIME

LINEAR SEQUENCES

• DESCRIBE IN WORDS

“STARTS WITH”
“GOES UP IN”

NEED 2
PIECES OF
INFORMATION

eg 3 7 11 15

STARTS WITH 3, GOES UP IN 4'S.

• WE USE LETTERS TO DESCRIBE THIS

a = START TERM

“STARTS WITH”

d = COMMON DIFFERENCE

“WHAT IT'S GOING UP IN”

“TERMS”

- ANY NUMBER IN THE SEQUENCE IS KNOWN AS A “TERM”

T_n → THIS IS HOW WE WRITE THE n^{th} TERM.

Q. WHAT IS THE n^{th} TERM?

A. IT'S A “MATHSY” WAY OF SAYING
A “GENERAL” TERM OR “ANY” TERM.

T_1 IS THE 1ST NUMBER IN THE PATTERN/SEQUENCE

T_2 IS THE 2ND “TERM”

T_7 IS THE 7TH “TERM” etc.

$T_n \rightarrow$ FORMULA FOR THE SEQUENCE

• WE ARE SOMETIMES GIVEN A FORMULA TO WORK OUT "ANY TERM" IN THE SEQUENCE T_n

$$T_1 = 1^{\text{ST}} \text{ "TERM"}$$

$$T_2 = 2^{\text{ND}} \text{ TERM ETC.}$$

eg $T_n = 2n + 5$

THIS IS A FORMULA FOR A PARTICULAR SEQUENCE.

$$T_1 = 2(1) + 5 = 7$$

SUBSTITUTE EACH NUMBER IN INSTEAD OF n .

THE 1ST THING TO DO IS WRITE OUT SOME TERMS IN THE SEQUENCE

$$T_2 = 2(2) + 5 = 9$$

$$T_3 = 2(3) + 5 = 11$$

NOW WE HAVE THE SEQUENCE

7, 9, 11, ...

BUT

$$T_{100} = 2(100) + 5 = 205$$

eg THE 100TH TERM IS 205

WE CAN USE T_n TO WORK OUT ANY TERM.

PATTERNS WITH PICTURES

STEP 1: WRITE OUT THE PATTERN

DESIGN	1	2	3	4
DOTS	1	3	5	7



WE HAVE TO TRY TO FIND THE PATTERN.

LOOK AT THE FIRST PICTURE AND COUNT THE NUMBER OF DOTS (IN OTHER QUESTIONS THIS COULD BE ANYTHING: SQUARES, MATCHSTICKS, CHAIRS etc...)

IT'S STARTING WITH 1 AND GOING UP IN 2'S.

HAVE A LOOK AT THE FOURTH TERM. THIS IS THE 1 ORIGINAL DOT, PLUS 3 LOTS OF TWO MORE DOTS.

$$T_4 = 1 + 3(2)$$

STARTING NUMBER DIFFERENCE

SIMILARLY

$$T_5 = 1 + 4(2) = 9$$

n n-1

So

$$T_n = 1 + (n-1)(2)$$

$$= 1 + 2n - 2$$

$$= \boxed{2n - 1}$$

THIS IS CALLED AN ARITHMETIC SEQUENCE...

THIS IS A SEQUENCE THAT HAS A STARTING

AMOUNT (USUALLY CALLED "a"), AND GOES

UP BY THE SAME DIFFERENCE (USUALLY CALLED "d")

EACH TERM.

WE COULD BE ASKED TO WORK BACKWARDS TO WORK OUT A FORMULA (T_n) FOR THE LINEAR SEQUENCE :

eg 7, 10, 13, 16, ...

1ST ANALYSE THE SEQUENCE :

THIS IS THE IMPORTANT INFO.

IT STARTS WITH 7 AND GOES UP IN 3s.

To WRITE T_n (FOR A LINEAR SEQUENCE)

WRITE $T_n = \text{---} n + \text{---}$

↑ WHAT IT'S GOING UP BY

↑ ADJUSTMENT TO GET "START TERM"

SO FOR OUR EXAMPLE

$$T_n = 3n + 4$$

↑
IT'S GOING UP IN 3s

$3 + 4 = 7$ ← START TERM
↑
ADJUSTMENT TO GET

MORE EXAMPLES (TRY COVERING THE ANSWERS AND DOING THESE YOURSELF)

① 6, 8, 10, 12, ...

$$T_n = 2n + 4$$

② 3, 7, 11, 15, ...

$$T_n = 4n - 1$$

③ 18, 15, 12, 9, ...

$$T_n = -3n + 21$$

PROBLEMS INVOLVING SEQUENCES

- ① WRITE OUT THE FIRST FEW AMOUNTS
- ② PUT THIS INTO A TABLE
- ③ DESCRIBE THE SEQUENCE :
IS IT — LINEAR
— QUADRATIC
— EXPONENTIAL .
- ④ DESCRIBE A RULE FOR GETTING THE NEXT TERM.
- ⑤ ALWAYS RELATE YOUR ANSWERS BACK TO THE QUESTION . DON'T FORGET UNITS .