PATTERNS + SEQUENCES

eg ① 3, 7, 11, 15 → GOING UP BY 4 EACH TIME.

② $3, 6, 11, 18$ with differences $+3$, $+5$, $+7$

→ NOW THE AMOUNT IT'S GOING UP BY IS GOING UP BY TWO EACH TIME.

③ 3, 6, 12, 24 → DOUBLING EACH TIME.

$\boxed{\text{FIND WHAT THE RULE IS!}}$

HAVE A LOOK AT THE <u>DIFFERENCE</u> BETWEEN EACH TERM AND THE NEXT.

"TERMS" – ANY NUMBER IN THE SEQUENCE IS KNOWN AS A "TERM"

T_n → THIS IS HOW WE WRITE THE n^{th} TERM.

Q. WHAT IS THE n^{th} TERM?

A. IT'S A "MATHSY" WAY OF SAYING A "GENERAL" TERM OR "ANY" TERM.

T_1 IS THE 1^{ST} NUMBER IN THE PATTERN/SEQUENCE

T_2 IS THE 2^{ND} "TERM"

T_7 IS THE 7^{TH} "TERM" etc.

TO FIND T_n, WE NEED TO KNOW THE "RULE" FOR THE PARTICULAR SEQUENCE.

WE ARE OFTEN TOLD WHAT T_n IS. WE CAN USE THIS TO CALCULATE ANY TERM.

eg $T_n = 3n + 1$

$T_1 = 3(1) + 1 = 4$

$T_2 = 3(2) + 1 = 7$

$T_3 = 3(3) + 1 = 10$

REPLACE THE n WITH WHATEVER TERM YOU ARE TRYING TO FIND

THIS COULD WORK BACKWARDS AS WELL:

eg FIND T_n FOR THE SEQUENCE $3,\ 7,\ 11,\ 15$

$\qquad\qquad +4 \quad +4 \quad +4$

YOU NEED TO FIND THE "DIFFERENCE" $\longrightarrow$

$\boxed{\text{THIS MEANS } \underline{T_n} \text{ WILL INCLUDE } \underline{4n}}$

TRY $T_n = 4n$

$T_1 = 4$

$T_2 = 8$

THESE ARE 1 TOO BIG, SO T_n MUST BE $4n - 1$

$T_n = 4n - 1$

$T_1 = 4(1) - 1 = 3$ ✓

$T_2 = 4(2) - 1 = 7$ ✓

$T_3 = 4(3) - 1 = 11$ ✓ etc.

<u>TO WRITE T_n</u>

SO, WRITE $T_n = \boxed{}\,n + \boxed{}$

<u>STEP 1</u>

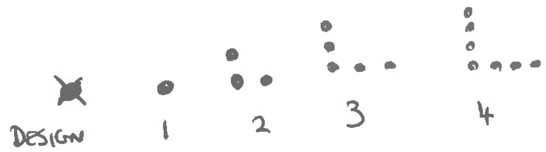
WHAT IT'S GOING UP BY

<u>STEP 2</u>

WHAT DO I HAVE TO ADD / SUBTRACT TO THE "GOING UP NUMBER" TO MAKE THE 1ST TERM OF THE SEQUENCE.

PATTERNS WITH PICTURES

eg



STEP 1: WRITE OUT THE PATTERN

DESIGN	1	2	3	4
DOTS	1	3	5	7

WE HAVE TO TRY TO FIND THE PATTERN.

LOOK AT THE FIRST PICTURE AND COUNT THE NUMBER OF DOTS (IN OTHER QUESTIONS THIS COULD BE ANYTHING: SQUARES, MATCHSTICKS, CHAIRS etc ...)

IT'S STARTING WITH 1 AND GOING UP IN 2's.

HAVE A LOOK AT THE FOURTH TERM. THIS IS THE 1 ORIGINAL DOT, PLUS 3 LOTS OF TWO MORE DOTS.

STARTING NUMBER

DIFFERENCE

$$T_4 = 1 + 3(2)$$

SIMILARLY $$T_5 = 1 + 4(2) = 9$$

n $\quad$ $n-1$

So

$$\begin{aligned} T_n &= 1 + (n-1)(2) \\ &= 1 + 2n - 2 \\ &= \boxed{2n - 1} \end{aligned}$$

THIS IS CALLED AN ARITHMETIC SEQUENCE ...

THIS IS A SEQUENCE THAT HAS A STARTING AMOUNT (USUALLY CALLED "a"), AND GOES UP BY THE SAME DIFFERENCE (USUALLY CALLED "d") EACH TERM.

ARITHMETIC SEQUENCES

eg $\quad 3 \underbrace{,\quad 7}_{+4} \underbrace{,\quad 11}_{+4} \underbrace{,\quad 15}_{+4} \; \ldots$

$a = 3 \qquad d = 4$

a = THE FIRST TERM

d = THE "DIFFERENCE". (OFTEN CALLED "COMMON DIFFERENCE")

FORMULA $\quad \boxed{T_n = a + (n-1)d}$

(IN TABLES BOOK)

THIS IS THE GENERAL FORMULA FOR "ANY" ARITHMETIC SEQUENCE. EACH SEQUENCE WILL HAVE ITS OWN PARTICULAR FORMULA, ONCE YOU SUBSTITUTE IN a AND d AND TIDY UP.

eg $\quad$ FIND A FORMULA FOR T_n FOR THE SEQUENCE

$2 \underbrace{,\ 5}_{+3} \underbrace{,\ 8}_{+3} \underbrace{,\ 11}_{+3}$

$a = 2$

$d = 3$

$$T_n = 2 + (n-1)3$$

$$= 2 + 3n - 3$$

$$\boxed{T_n = 3n - 1}$$

FINDING a AND d.

IF IT MENTIONS THE 5^{TH} TERM, YOU SHOULD IMMEDIATELY BE THINKING $a + 4d$

IF IT MENTIONS T_{11}, YOU THINK / WRITE $a + 10d$

SAME FOR ANY TERM

IF YOU DO THIS, MANY QUESTIONS BECOME VERY EASY. ALTHOUGH YOU WILL PROBABLY HAVE TO SOLVE SIMULTANEOUS EQUATIONS.

TRICK. IF IT GIVES YOU AN ARITHMETIC SEQUENCE WITH A WHOLE LOAD OF x's (OR FOR ANY ARITHMETIC SEQUENCE), REMEMBER:

$$\boxed{T_2 - T_1 = T_3 - T_2}$$

BECAUSE
$T_2 - T_1 = d$
$T_3 - T_2 = d$

THIS CAN GET A BIT MESSY, BUT THEY SHOULDN'T MAKE THE EQUATION **TOO** DIFFICULT.

eg $\underset{T_1}{x+1}, \underset{T_2}{2x-2}, \underset{T_3}{2x+1}$ ARE THE 1^{ST} 3 TERMS OF AN ARITHMETIC SEQUENCE. FIND x.

So

$$T_2 - T_1 = T_3 - T_2$$
$$(2x-2) - (x+1) = (2x+1) - (2x-2)$$
$$2x - 2 - x - 1 = \cancel{2x} + 1 - \cancel{2x} + 2$$
$$x - 3 = 3$$
$$+3 \qquad +3$$
$$\boxed{x = 6}$$

So $T_1 = 6 + 1 = 7$
$T_2 = 2(6) - 2 = 10$
$T_3 = 2(6) + 1 = 13$

7, 10, 13, ...

WHEN WE <u>ADD</u> TOGETHER TERMS OF AN ARITHMETIC SEQUENCE, MATHEMATICIANS CALL THIS A $\boxed{\text{SERIES}}$

WE USE S_n (A BIT LIKE T_n – EXCEPT IT'S THE "SUM" OF ANY NUMBER OF TERMS)

eg 1, 3, 5, 7

T_1 = THE FIRST TERM = 1

S_1 = THE <u>SUM</u> OF THE FIRST ONE TERMS = 1

$T_2 = 3$

$S_2 = 1 + 3 = 4$

$S_3 = 1 + 3 + 5 = 9$

THERE IS ANOTHER FORMULA (SURPRISE SURPRISE!)

$$\boxed{S_n = \frac{n}{2}\left(2a + (n-1)d\right)}$$

- IT CAN BE VERY HELPFUL TO SUBTRACT CONSECUTIVE "SUMS" OF SERIES, TO FIND A PARTICULAR TERM. <u>THINK</u> ABOUT THE FOLLOWING:

eg S_5 = SUM OF THE FIRST 5 TERMS

S_4 = SUM OF THE FIRST 4 TERMS

THE ONLY DIFFERENCE BETWEEN THESE IS THE 5^{TH} TERM!

SO,

$$\boxed{S_5 - S_4 = T_5}$$

OTHER SEQUENCES

WE USED TO HAVE TO KNOW ABOUT GEOMETRIC SEQUENCES

eg $\underbrace{2, 4}_{\times 2}, \underbrace{8}_{\times 2}, \underbrace{16}_{\times 2}$ etc.

WE DO NEED TO KNOW ABOUT

QUADRATIC SEQUENCES — UGH!

BASICALLY, IF IT'S NOT AN ARITHMETIC SEQUENCE, IT MIGHT WELL BE ONE OF THESE...

HOW DO YOU KNOW?

SO YOU'RE GIVEN A SEQUENCE $1, 4, 9, 16, \ldots$

TRY TO FIND THE COMMON DIFFERENCE → $+3 \quad +5 \quad +7$

$+2 \quad +2$

(HMM DO YOU NOTICE ANYTHING)

YES...

THIS IS CALLED THE "SECOND DIFFERENCE". THIS IS A QUADRATIC SEQUENCE.

$$T_n = an^2 + bn + c$$

WE JUST HAVE TO REMEMBER THIS (SORRY)

a IS ALWAYS $\frac{1}{2}$ THE "SECOND DIFFERENCE"

YOU HAVE TO USE SIMULTANEOUS EQUATIONS TO FIND b AND c

eg $3 \quad 10 \quad 21 \quad 36$ FIND T_n.

$7 \quad 11 \quad 15$ ← 1^{st} DIFFERENCE

$4 \quad 4$ ← 2^{nd} DIFFERENCE.

$$T_n = an^2 + bn + c$$

BUT $a = \frac{1}{2}$ OF 4

$a = 2$

$$T_n = 2n^2 + bn + c$$

WE KNOW

$T_1 = 3$

$2(1)^2 + b(1) + c = 3$

$\underset{-2}{2} + b + c = \underset{-2}{3}$

$b + c = 1$

AND

$T_2 = 10$

$2(2)^2 + b(2) + c = 10$

$\underset{-8}{8} + 2b + c = \overset{-8}{10}$

$2b + c = 2$

CHANGE SIGNS → $-b - \cancel{c} = -1$

$2b + \cancel{c} = 2$

ADD

$b = 1$

$b + c = 1$

$1 + c = 1$

$c = 0$

So $T_n = 2n^2 + 1n + 0$

$$\boxed{T_n = 2n^2 + n}$$