PROBABILITY (CHANCE OF SOMETHING HAPPENING)

NON-NUMERICAL PROBABILITY / LIKELIHOOD : WE USE
- IMPOSSIBLE
- CERTAIN
- LIKELY
- UNLIKELY
- EVEN CHANCE

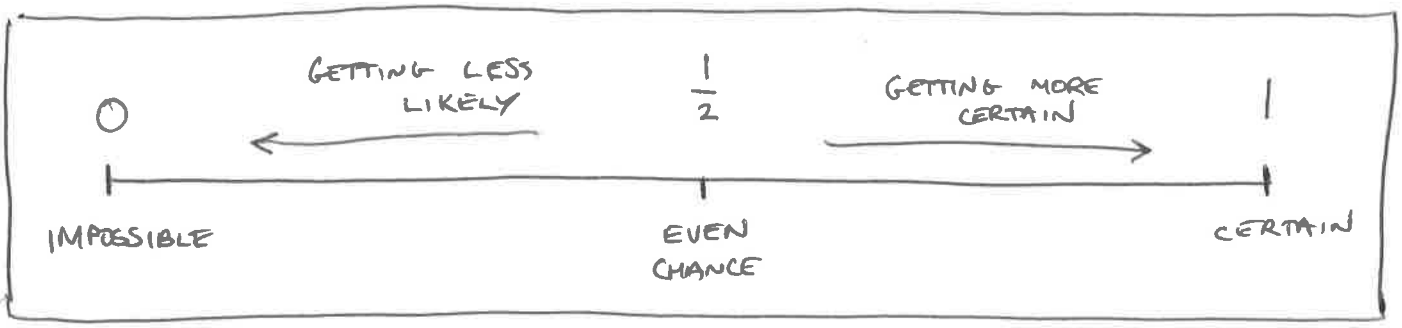
etc.

NOTE : EVEN CHANCE DOES <u>NOT</u> MEAN THE SAME THING AS EQUALLY LIKELY !

eg THE PROBABILITY OF GETTING A "HEAD" FROM THE TOSS OF A COIN <u>IS</u> EVEN CHANCE. (OR 50/50)

BUT THE PROBABILITY OF A DICE LANDING ON A 3 IS <u>NOT</u> EVEN CHANCE – EVEN THOUGH IT IS <u>EQUALLY LIKELY</u> TO LAND ON A 3 AS ANY OTHER NUMBER.

WE MEASURE PROBABILITY OR LIKELIHOOD ON A SCALE FROM 0 (IMPOSSIBLE) TO 1 (CERTAIN)



<u>PROBABILITY</u> IS <u>NUMERICAL</u>. IT MUST BE BETWEEN 0 AND 1. WE USUALLY WRITE PROBABILITY AS A FRACTION (BUT IT IS OFTEN WRITTEN AS A DECIMAL TOO).

$$\boxed{\text{PROBABILITY} = \frac{\text{NUMBER OF DESIRABLE OUTCOMES}}{\text{TOTAL NUMBER OF POSSIBLE OUTCOMES}}}$$

THIS IS THE NUMBER OF OUTCOMES WHICH MATCH WHATEVER WE'RE CALCULATING THE PROBABILITY OF

eg PROBABILITY OF SELECTING A "HEART" FROM A DECK OF CARDS $= \frac{13}{52}$

13 ← NUMBER OF HEARTS

52 ← TOTAL NUMBER OF CARDS

(FOR MORE ON "OUTCOMES" SEE NOTES ON COUNTING / PERMUTATIONS)

RELATIVE FREQUENCY – ALSO KNOWN AS EXPERIMENTAL PROBABILITY

- BASED ON OBSERVATIONS / TRIALS OF ACTUAL EVENTS (eg ROLLING A DICE / SCORING A PENALTY etc)

$$\boxed{\text{RELATIVE FREQUENCY} = \frac{\text{NUMBER OF TIMES THAT EVENT HAPPENS}}{\text{NUMBER OF TRIALS CARRIED OUT}}}$$

eg. IF I ROLL A DICE 100 TIMES AND GET A "SIX" 20 TIMES, THEN THE RELATIVE FREQUENCY OF GETTING A SIX IS $\boxed{\frac{20}{100}}$ (WE SAY 20 "OUT OF" 100)

IMPORTANT

THE MORE TIMES WE CARRY OUT THE EXPERIMENT / TRIALS, THE MORE RELIABLE THE RELATIVE FREQUENCY BECOMES.

SO IF I TOSS A COIN FIVE TIMES AND GET FOUR TAILS, IT DOESN'T MEAN IT'S A BIASED COIN, BUT IF I TOSS IT 5,000 TIMES AND GET 4,000 TAILS, WE CAN PROBABLY SUGGEST THE COIN MUST BE BIASED!

EXPECTED FREQUENCY – HOW MANY TIMES WOULD WE EXPECT A CERTAIN EVENT HAPPENS?

ANSWER: MULTIPLY THE NUMBER OF TRIALS BY THE PROBABILITY OR THE RELATIVE FREQUENCY.

eg IF A SPINNER LANDS ON RED 50 TIMES OUT OF 75 SPINS, HOW MANY TIMES SHOULD IT LAND ON RED IF I SPIN IT 200 TIMES

$$\text{EXPECTED FREQUENCY} = 200 \times \underbrace{\frac{50}{75}}_{\text{RELATIVE FREQUENCY}}$$

SINGLE EVENTS (AND / OR)

eg Q. WHAT IS THE PROBABILITY OF GETTING A KING OR A QUEEN FROM A DECK OF CARDS?

A. THERE ARE 4 KINGS AND 4 QUEENS. SO THE PROBABILITY IS $\frac{8}{52}$

OR RULE : ADD THE PROBABILITIES.

OR ⇒ ADD.

BE CAREFUL : YOU MUST SUBTRACT THE PROBABILITY OF BOTH OUTCOMES

eg. Q. WHAT IS THE PROBABILITY OF GETTING A KING OR A CLUB

A. = $\underset{\text{CLUBS}}{\frac{13}{52}} + \underset{\text{KINGS}}{\frac{4}{52}} - \frac{1}{52}$

WE MUST SUBTRACT THE NUMBER WHICH ARE KINGS + CLUBS i.e. KING OF CLUBS

MUTUALLY EXCLUSIVE EVENTS : HAVE NO OUTCOMES IN COMMON

eg ① PROBABILITY OF GETTING A KING OR QUEEN ?

THERE IS NO CARD WHICH IS BOTH A KING AND A QUEEN,

SO WE CALL THESE EVENTS **MUTUALLY EXCLUSIVE**

② PROBABILITY OF GETTING A KING OR A CLUB.

THESE ARE NOT MUTUALLY EXCLUSIVE, AS YOU CAN GET THE **KING OF CLUBS**

~~NOT~~ THERE ARE OUTCOMES IN COMMON.

"AND" EVENTS

eg Q WHAT IS THE PROBABILITY OF GETTING A KING AND A HEART ?

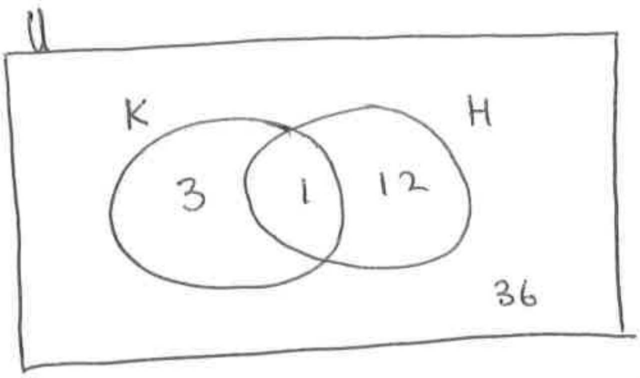
A. HOW MANY CARDS ARE A HEART AND A KING ? THERE IS JUST 1, THE KING OF HEARTS

$\Rightarrow$ THE PROBABILITY OF GETTING A KING AND A HEART IS $\frac{1}{52}$

VENN DIAGRAMS (SETS)

- THESE CAN BE VERY USEFUL FOR CALCULATING PROBABILITIES SUCH AS AND / OR

eg THE EXAMPLE ABOVE



$\# U$ = TOTAL NUMBER OF CARDS

$\# K$ = NUMBER OF KINGS

$\# H$ = NUMBER OF HEARTS.

eg (i) Q. WHAT'S THE PROBABILITY OF GETTING A KING OR A HEART ?

A. $3 + 1 + 12 = 16$, SO $\boxed{\frac{16}{52}}$

(ii) Q. WHAT'S THE PROBABILITY OF GETTING A KING AND A HEART

A. $\frac{1}{52}$ (THE "INTERSECTION")

COMBINED EVENTS (MORE THAN ONE EVENT HAPPENS)

1) USING TREE DIAGRAM / TWO-WAY TABLE / SYSTEMATIC LISTING, WRITE DOWN ALL THE COMBINED OUTCOMES WHICH ARE POSSIBLE.

2) USE SAME RULES OF PROBABILITY AS BEFORE.
i.e. HOW MANY COMBINED OUTCOMES MATCH WHAT WE'RE LOOKING FOR ?

MULTIPLE EVENTS

- INDEPENDENT
- NOT INDEPENDENT

AND	⇒	×
OR	⇒	+

YOU MUST BE ABLE TO USE THESE RULES.

eg... A WHITE DICE AND A BLACK DICE ARE THROWN.
FIND THE PROBABILITY OF GETTING :

(i) 2 ON THE WHITE AND 4 ON THE BLACK.
(ii) 2 ON THE WHITE AND ANY NUMBER ON THE BLACK.
(iii) A 2 ON ONE DICE AND A 4 ON THE OTHER...

(i) p(2 ~~RED~~ WHITE) AND p(4 BLACK)

$$\frac{1}{6} \times \frac{1}{6} = \boxed{\frac{1}{36}}$$

(ii) p(2 WHITE) AND p(ANY BLACK)

$$\frac{1}{6} \times \frac{6}{6} = \frac{6}{36} = \boxed{\frac{1}{6}}$$

(iii) THIS COULD BE (2 WHITE AND 4 BLACK) OR (4 WHITE AND 2 BLACK)

SO, $\frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6}$

$$= \frac{1}{36} + \frac{1}{36} = \boxed{\frac{2}{36}} = \boxed{\frac{1}{18}}$$

BERNOULLI TRIALS

THIS IS REALLY JUST THE SAME AS LOTS OF MULTIPLE EVENTS, BUT WHERE THERE ARE ONLY 2 POSSIBLE OUTCOMES,

$\boxed{\text{SUCCESS}}$ OR $\boxed{\text{FAIL}}$

eg FLIPPING A COIN, WHEN "TAILS" IS A SUCCESS.

Q WHAT'S THE PROBABILITY THAT YOU GET THE FIRST "TAILS" ON THE THIRD FLIP?

A. TO DO THIS, YOU NEED TO WORK OUT WHAT IS THE PROBABILITY OF GETTING A HEAD, THEN ANOTHER HEAD, THEN A TAIL...

SO $p(H) \times p(H) \times p(T)$

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

HARDER EXAMPLE...

A SPINNER WITH EQUAL SECTORS NUMBERED 1 TO 5 IS SPUN. WHAT IS THE PROBABILITY THAT IT LANDS ON AN ODD NUMBER FOR THE FIRST TIME ON THE:

(i) FIRST SPIN (ii) SECOND SPIN (iii) THIRD SPIN.

(i) $p(\text{ODD}) = \frac{3}{5}$

(ii) $p(\text{EVEN})$ AND THEN $p(\text{ODD})$

$$\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$$

(iii) $p(\text{EVEN THEN EVEN THEN ODD})$

$$= \frac{2}{5} \times \frac{2}{5} \times \frac{3}{5} = \frac{12}{125}$$

EXPECTED VALUE is the "average" outcome of an experiment / game...

To calculate the expected value,

multiply each outcome amount by the probability of that outcome, and add all your answers together.

$$\boxed{\text{EXPECTED VALUE} = \text{ADD ALL OUTCOME} \times \text{PROBABILITY}}$$

eg Q. A die is rolled. If a six is scored, I win €10
If a five is scored, I win €3
If anything else, I win nothing.
What is the expected value of this game?

A.

	OUTCOME		PROBABILITY		VALUE
6.	€10	×	$\frac{1}{6}$	=	1.67
5.	€3	×	$\frac{1}{6}$	=	0.50
1 → 4	€0	×	$\frac{4}{6}$	=	0

$$\boxed{\text{EXPECTED VALUE} = €2.17}$$

If this game cost €2 to play, I would expect to win 17c on average