Leaving Cert - Revision Sheet 2

Algebra

1. Solve the following simultaneous equations

\[ 3x - 2y = 19 \]
\[ 2x + 3y = 30 \]

2. Solve the equation

\[ \frac{2x}{3} - \frac{x}{4} = \frac{5}{2} \]
Co-ordinate Geometry

3. Investigate which of the two points A(3, −2) or B(4, −1) is on the line \( x + 2y + 1 = 0 \)

4. Write down the equation of the line which has slope \( \frac{3}{4} \) and crosses the y-axis at (0, -2)

5. Write down the equation of the line which has slope 2 and passes through the point \((-3, 7)\)

Statistics

6. Calculate the mean, median, and mode of the following list of numbers:
   \[ 11, 24, 13, 25, 87, 9, 11, 23, 7, 11 \]

7. Which average do you think is the most appropriate to use for the list of numbers above. Give a reason for your answer.
Patterns / Sequences

8. 
4, 7, 10, 13, ...

Which term of the given arithmetic sequence is 127?

9. In an arithmetic sequence, \( T_4 = 14 \) and \( T_9 = 34 \)
   (a) Find the values of a (the first term), and d (the common difference)

   (b) Write down \( T_n \) for this sequence

   (c) Calculate \( T_{60} \)
10. If there is a complex number, \( z = 3 + 4i \):
   Calculate
   (a) \( \bar{z} \)
   (b) \(|z|\)

11. A company borrowed €12,000 from a bank at 8% per annum compound interest.
   Assuming that they make no repayments, how much will they owe at the end of 3 years?

12. A car depreciates in value by 20% per year. If the car cost €25,000 brand new, how much is it worth after 6 years?

13. An investment bond gives an overall return of 25% after 10 years. Calculate the AER (annual equivalent rate) for this bond. (Hint: Assume that a certain amount is invested e.g. €10,000)
Probability

14. John thinks his coin is biased. He tosses it 600 times and gets 420 heads and 180 tails.
(a) What is the relative frequency of getting a head with his coin?

(b) In 600 tosses, how many heads would you expect to get?

(c) Do you think his coin is biased? Explain your answer.

15. A bag contains 5 Red discs and 4 Blue discs. A disc is selected at random and then replaced. A second disc is then selected. Find the probability that:
(a) Both discs are red

(b) The first is red and the second is blue

(c) The first is blue and the second is red

(d) They are both the same colour.

Geometry

16. $[AB]$ is the diameter of a circle with centre O. $[CD]$ is a chord with a midpoint M. $AB \perp CD$

$|CD| = 8 \text{ cm}$ and $|OM| = 3 \text{ cm}$
(a) Find $|OC|$, the radius of the circle

(b) Find $|MB|$

(c) What is $|\angle ACB|$? Give a reason for your answer.

(d) Calculate the area of the triangle ABC.
1. Solve the following simultaneous equations

\[
\begin{align*}
\text{A} & \quad 3x - 2y = 19 \\
\text{B} & \quad 2x + 3y = 30 \\
\text{C} & \quad 9x - 6y = 57 \\
\text{D} & \quad 4x + 6y = 60
\end{align*}
\]

\[
\begin{align*}
13x & = 117 \\
x & = 9
\end{align*}
\]

2. Solve the equation

\[
\frac{2x}{3} - \frac{x}{4} = \frac{5}{2}
\]

\[
\sqrt[4]{\left(\frac{2x}{3}\right)} - \sqrt[3]{\left(\frac{x}{4}\right)} = \sqrt[5]{\left(\frac{5}{2}\right)}
\]

\[
8x - 3x = 30
\]

\[
x = 6
\]
Co-ordinate Geometry

3. Investigate which of the two points A(3, -2) or B(4, -1) is on the line $x + 2y + 1 = 0$

   A(3, -2) $\rightarrow$ $(3) + 2(-2) + 1 = 0 \quad \checkmark \quad \text{ON THE LINE}$

   B(4, -1) $\rightarrow$ $(4) + 2(-1) + 1 = 0 \quad \times$

4. Write down the equation of the line which has slope $\frac{3}{4}$ and crosses the y-axis at (0, -2)

   \[ y = \frac{3}{4}x - 2 \]

5. Write down the equation of the line which has slope 2 and passes through the point (-3, 7)

   \[ y - y_1 = m(x - x_1) \]

   \[ y - 7 = 2(x + 3) \]

Statistics

6. Calculate the mean, median, and mode of the following list of numbers:

   \[ 11, 24, 13, 25, 87, 9, 11, 23, 7, 11 \]

   \[
   \text{mean} = \frac{\text{Sum}}{\text{Num}} = 22.1
   \]

   \[
   \text{mode} = 11
   \]

   \[
   \text{median} = 12
   \]

7. Which average do you think is the most appropriate to use for the list of numbers above.
   Give a reason for your answer.

   Median - not affected by the outlier/ extreme value.
Patterns / Sequences

Which term of the given arithmetic sequence is 127?

\[ T_n = 3n + \frac{1}{4} = 127 \]

\[ 3n = 126 \]
\[ n = 42 \]

42\textsuperscript{nd} Term = 127

9. In an arithmetic sequence, \( T_4 = 14 \) and \( T_9 = 34 \)
   
   (a) Find the values of \( a \) (the first term), and \( d \) (the common difference)

   \[ \begin{array}{c}
   T_1 \quad T_2 \quad T_3 \quad T_4 \quad T_5 \quad T_6 \quad T_7 \quad T_8 \quad T_9 \\
   2 \quad 6 \quad 10 \quad 14 \quad 18 \quad 22 \quad 26 \quad 30 \quad 34 \\
   \end{array} \]

   \[ d = 4 \]

   \[ a = 2 \]

   (b) Write down \( T_n \) for this sequence

   \[ T_n = 4n - 2 \]

   (c) Calculate \( T_{60} \)

   \[ T_{60} = 4(60) - 2 \]
   \[ = 238 \]
10. If there is a complex number, \( z = 3 + 4i \):
Calculate
(a) \( \bar{z} \), the conjugate of \( z \)
\[ \bar{z} = 3 - 4i \]
(b) \( |z| \), the modulus or size
\[ |z|^2 = 3^2 + 4^2 \]
\[ |z|^2 = 25 \]
\[ |z| = 5 \]

11. A company borrowed €12,000 from a bank at 8% per annum compound interest.
Assuming that they make no repayments, how much will they owe at the end of 3 years?
\[ F = P \left(1 + \frac{r}{n}\right)^{nt} \]
\[ = 12000 \left(1 + 0.08\right)^3 \]
\[ = €15116.54 \]

12. A car depreciates in value by 20% per year. If the car cost €25,000 brand new, how much is it worth after 6 years?
\[ F = P \left(1 - \frac{d}{100}\right)^t \]
\[ = 25000 \left(1 - 0.20\right)^6 \]
\[ = €6553.60 \]

13. An investment bond gives an overall return of 25% after 10 years. Calculate the AER (annual equivalent rate) for this bond. (Hint: Assume that a certain amount is invested e.g. €10,000)
\[ F = P \left(1 + \frac{r}{100}\right)^t \]
\[ = 12500 \]
\[ \frac{10000}{10000} \left(1 + \frac{r}{100}\right)^{10} = \frac{12500}{10000} \]
\[ (1 + \frac{r}{100})^{10} = 1.25 \]
\[ 1 + \frac{r}{100} = 10^{\sqrt{1.25}} \]
Probability

14. John thinks his coin is biased. He tosses it 600 times and gets 420 heads and 180 tails.
   (a) What is the relative frequency of getting a head with his coin?
      \[
      \frac{420}{600} = \frac{7}{10} = 0.7
      \]
   (b) In 600 tosses, how many heads would you expect to get? If it was a fair coin
      \[
      \frac{1}{2} \times 600 = 300
      \]
   (c) Do you think his coin is biased? Explain your answer.
      YES. I would expect the number of heads to be closer to 300. 600 is a large sample size

15. A bag contains 5 Red discs and 4 Blue discs. A disc is selected at random and then replaced. A second disc is then selected. Find the probability that:
   (a) Both discs are red
      \[
      \frac{5}{9} \times \frac{5}{9} = \frac{25}{81}
      \]
   (b) The first is red and the second is blue
      \[
      \frac{5}{9} \times \frac{4}{9} = \frac{20}{81}
      \]
   (c) The first is blue and the second is red
      \[
      \frac{4}{9} \times \frac{5}{9} = \frac{20}{81}
      \]
   (d) They are both the same colour.
      \[
      \frac{5}{9} \times \frac{5}{9} = \frac{25}{81}
      \]

Geometry

16. \([AB]\) is the diameter of a circle with centre O. \([CD]\) is a chord with a midpoint M. AB \perp CD

   \(|CD| = 8 \text{ cm} \text{ and } |OM| = 3 \text{ cm}\)
(a) Find $|OC|$, the radius of the circle

\[ r^2 = 3^2 + 4^2 \]
\[ r^2 = 9 + 16 \]
\[ r^2 = 25 \]
\[ r = 5 \]

(b) Find $|MB|$

\[ 5 - 3 = 2 \]

(c) What is $|\angle ACB|$? Give a reason for your answer.

90° $\Rightarrow$ ANGLE IN A SEMI-CIRCLE IS

- ANGLE IN A RIGHT
- OPPOSITE DIAMETER

(d) Calculate the area of the triangle ABC.

\[ A = \frac{1}{2}bh \]
\[ A = \frac{1}{2} \times 10 \times 4 \]
\[ A = 20 \]